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Abstract

This paper focuses on the weak separability of various monetary aggregates, narrow or broad as the DM4 one. Since weak separability is both a necessary and sufficient condition for aggregating over monetary assets, our paper has straightforward implications in terms of monetary measurement. It also returns key information about the definition of money. To test for weak separability, we implement the Barnett and de Peretti (2009) semi-nonparametric test for weak separability, partly based on the General Axiom of Reveal Preference. With regard to existing competing tests, their procedure is both: i) Necessary and sufficient, ii) Stochastic, i.e. dealing with measurement error in quantity data or small optimization error. Moreover, in this paper, we extend the test to also deal with incomplete adjustment, and introduce a new empirical econometric procedure to test for the separability condition, robust to measurement errors. Using monthly US data, spanning 1968:01 to 2012:12, our results support the weak separability of a broad monetary aggregate as DM4 or DM4−.

1 Introduction

This paper uses a semi non-parametric and stochastic approach to the determination of admissible clusters of recently developed broad Divisia monetary aggregates for the United States. The Center for Financial Stability began providing broad Divisia monetary aggregates in 2011, with clustering based on the traditional M1, M2, M3, and M4 levels used by the Federal Reserve Board and the Bank of England. The narrow (M1, M2) versions1 of these new monetary

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1Other narrow versions of the Divisia monetary aggregates are provided by the St. Louis Federal Reserve Board and are computed with a different benchmark rate. See Anderson and Jones (2011) for the details.
aggregates already serve as the basis for new work in applied macroeconomics and macroeconomic theory such as Belongia and Ireland (2011, 2013), Serletis and Gogas (2013), and the broad measures have been used in policy papers on the state of the US economy released by Hanke (2011).

The clustering however is not based in empirical evidence but the traditional distinction between “liquid” and “not liquid” assets. There is a vast and well established literature on both parametric and non parametric methods for empirically determining the proper clustering and existence of components within and of an aggregate based on the utility maximization of a representative agent. We separate from the parametric branch of this literature in order to avoid specifying a functional form, and instead focus on the non-parametric literature for admissibility and weak separability based on Varian (1982) and Afriat (1967, 1973). The advantageous nature of the non-parametric approach stems from its use of the observed data rather than a possibly incorrect specification of a functional form for the demand function. The non parametric approach allows the data to speak.

The data however are not perfectly observed; there exists noise in data due to measurement error or other stochastic shocks. The empirical test for weak separability therefore has a tendency to over reject based on not significant measurement error or other shocks, as demonstrated through Monte-Carlo simulations in Barnett and Choi (1998). The methodology used here accounts for these stochastic shocks based on the work of Barnett and de Peretti (2009) in order to determine if the rejection of weak separability is warranted based on factors other than measurement error and stochasticity.

A further improvement on weak separability testing is used that provides a necessary and sufficient test based on microeconomic foundations of the relationship between prices and the marginal rate of substitution. This relationship is used in Barnett and de Peretti (2009) to develop a straightforward test without use of Afriat inequalities to test the relationship of prices within a cluster to changes in the levels outside a cluster.

The paper will test the broad Divisia aggregates for admissibility through these new methods that account for stochastic shocks and over rejection. In section 2 the theoretical foundation of the aggregates used and the Varian (1982) test of the Generalized Axiom of Revealed Preference, along with the Barnett and de Peretti (2009) extension are explained. Section 3 provides a detailed discussion of the data used in the preliminary results and the more comprehensive data set to be used on the improvement of this project. Section 4 reviews the results of the admissibility tests. Section 5 concludes with possible extensions in empirical work.
2 Theoretical Foundations

2.1 Aggregation Theory

The Divisia monetary aggregates in this paper and provided by the Center for Financial Stability (CFS) are based on the seminal work of Barnett (1980) in monetary aggregation and Barnett (1978) and Offenbacher and Schachar (2011) for user cost pricing of monetary services.

To begin, money is treated as a durable good as defined formally in Diewert (1974), meaning that the proper pricing of monetary service is similar to a durable good; it is priced by its rental price, or user cost. Barnett (1978) derived the user cost of money for

\[ R_B^i = 1 + r_B^i \]

the return on a benchmark asset with an interest rate \( r_B^i \) and \( R_{ij} = 1 + r_{ij} \) the return on monetary component \( j \) at observation \( i \) with interest rate \( r_{ij} \). The user cost is defined as:

\[ \pi_{ij} = \frac{R_B^i - R_{ij}}{R_B^i} \]  

(1)

For the CFS data the benchmark rate chosen is the short term rate for bank loans to commercial and industrial customers. This rate approximates the highest possible return that a bank would provide as the bank would not offer to pay more on interest than it loans out. Using this method described in Offenbacher and Schachar (2011) for the Bank of Israel Divisia monetary aggregates avoids the use of an arbitrary addition of basis points to a highest rate of return chosen from the basket of monetary asset returns.

Given the proper pricing of monetary assets, one can then solve for the expenditure share on asset \( j \) at observation \( i \) over the range of monetary assets included in the aggregate. For some optimal level of monetary assets \( m_{ij}^* \) ranging from \( \ell = 1, ..., j, ... L \) the expenditure share is defined as:

\[ s_{ij} = \frac{\pi_{ij} m_{ij}^*}{\sum_{l=1}^{L} \pi_{il} m_{il}^*} \]  

(2)

The Divisia monetary aggregate at observation \( i \), \( M_i \) can then be solved through the Tornqvist-Theil approximation:

\[ \log(M_i) - \log(M_{i-1}) = \sum_j \bar{s}_{ij} \left( \log(m_{ij}^*) - \log(m_{i-1,j}^*) \right) \].  

(3)

In this case \( \bar{s}_{ij} = s_{ij} + s_{i-1,j} / 2 \). After indexing the base year\(^2\) to 100, the Divisia growth rates can be solved and used in analysis.

2.2 Weak Separability Testing

2.2.1 Generalized Axiom of Revealed Preference

As defined in Varian (1982) the Generalized Axiom of Revealed preference (GARP) determines the proper clustering through weak separability. For \( i =

\(^2\)For the CFS and for this paper the base month and year are 1967:01.
1, ..., T observations and k = 1, ..., K quantity components, let $X$ be the $(T \times K)$ matrix of real per capita quantities. The $i$th rows of $X$ are $x_i = (x_{i1}, x_{i2}, \ldots, x_{iK})$. The price matrix and its corresponding rows are $P$ and $p_i = (p_{i1}, p_{i2}, \ldots, p_{iK})$. Partitioning out the components to be tested for weak separability, let $X^{(1)}$ be a $(T \times a)$ matrix with $X^{(2)}$ the $(T \times (k-a))$ matrix of quantities outside the hypothetical weakly separable group. Corresponding partitions of the prices matrices are denoted as $P^{(1)}$ and $P^{(2)}$.

**Weak Separability Testing 1.** If there exists an overall utility function $U(\cdot)$, a strictly increasing macro function $V(\cdot)$, and a sub-utility function $f(\cdot)$ which can be represented as,

$$U_i = U(x_i),$$  \hspace{1cm} (4)  

$$U_i = V(x_{i2}^{(2)}, f(x_{i1}^{(1)})), $$  \hspace{1cm} (5)  

then $X^{(1)}$ is weakly separable from the outside group components $X^{(2)}$.

It follows that the marginal rate of substitution of components within the weakly separable group are independent of changes in the outside group. For components $j, l = 1, \ldots, a$ and $j \neq l$, $m = (a + 1), \ldots, k$ and $i = 1, \ldots, T$:

$$\frac{\partial U(x_i)}{\partial x_{ij}} \frac{\partial U(x_i)}{\partial x_{il}} = 0$$  \hspace{1cm} (6)  

The nonparametric literature that begins with Varian (1982, 1983) checks the existence of the overall utility, the macrofunction, and sub-utility function using GARP, but does not use this relationship between the marginal rates of substitution. If an overall aggregate of the observed data set $\{(x_i, p_i)\}_{i=1}^T$ is to exist (have an overall and sub-utility function) then the data must behave in a utility maximizing way, and hence must satisfy GARP. The Varian (1982) procedure begins with testing the data for satisfying GARP. Before defining GARP, the binary relations must be defined and reviewed, namely:

i $x_i, P^0 x_j$, the bundle $x_i$ is strictly directly revealed preferred to $x_j$ if $\forall i, j \in (1, \ldots, T)$,  

$$p_i \cdot x_i > p_j \cdot x_j$$  \hspace{1cm} (7)  

ii $x_i, R^0 x_j$, the bundle $x_i$ is directly revealed preferred to $x_j$ if $\forall i, j \in (1, \ldots, T)$,  

$$p_i \cdot x_i \geq p_j \cdot x_j$$  \hspace{1cm} (8)  

iii $x_i, R x_j$, the bundle $x_i$ is revealed preferred to $x_j$ if $\forall i, j \in (1, \ldots, T)$ there is a transitive closure between the observations,  

$$x_i, R^0 x_m, x_m, R^0 x_n, \ldots, x_p, R^0 x_j$$  \hspace{1cm} (9)  

or  

$$p_i \cdot x_i \geq p_i \cdot x_m, p_m \cdot x_m \geq p_n \cdot x_n, \ldots, p_p \cdot x_p \geq p_p \cdot x_j$$  \hspace{1cm} (10)  

4
Given these relations GARP is defined in two possible ways:

**GARP 1. Binary Preference Definition.** The observed set \( \{(x_i, p_i)\}_{i=1}^T \) satisfies the Generalized Axiom of Revealed Preference for \( i \) and \( j \) if the revealed preference relation \( x_i R x_j \) implies that the \( j \) is not strictly directly revealed preferred to \( i \), “not \( x_j P x_i \).”

**GARP 1. Inequality Definition.** The observed set \( \{(x_i, p_i)\}_{i=1}^T \) satisfies the Generalized Axiom of Revealed Preference for \( i \) and \( j \) if \( p_i \cdot x_i \geq p_i \cdot x_m, p_m \cdot x_m > p_m \cdot x_n, ..., p_p \cdot x_p > p_p \cdot x_j \) implies \( p_j \cdot x_j \leq p_j \cdot x_i \).

Using GARP, the literature based on Varian (1982) develops non-parametric conditions for testing weak separability and the existence of the overall and sub-utility function. The weakness in the literature lies in the necessary but not sufficient nature of these conditions; while GARP is a necessary condition for the existence of theoretically consistent aggregate, it is not sufficient. Further the third condition using Afriat Inequalities found in Varian (1982) and its continuing literature is a sufficient but not necessary condition. The Barnett and de Peretti (2009) method provides a necessary and sufficient condition complementing the initial necessary conditions of satisfying GARP.

### 2.2.2 Weak Separability

Following the weak separability theorem in Varian (1982) and the marginal rate of substitution relationship defined in Equation (6) Barnett and de Peretti (2009) formulate a necessary and sufficient weak separability test using price ratios. The test is straightforward: first determine if the overall data set satisfy GARP, then determine if the weakly separable group satisfies GARP, then econometrically determine if the price ratios of the components in the weakly separable group are independent from changes in quantity of the outside components. In order to test this third condition, the price ratios of the weakly separable components are checked for dependence on the partitioned matrix of outside components. A simple regression is run on the price ratios using the component quantities and if the coefficients of the outside group quantities are not significantly different from zero then there is no evidence of dependence outside the cluster, GARP is satisfied, the utility functions behave in an optimizing fashion, and an aggregate exists.

Expand the quantity matrix \( X^{(3)} = [1log(X)] = [1log(X^{(1)})log(X^{(2)})] \) to include a matrix of ones to account for a constant and partition the weak separable cluster to be tested from the outside quantities. \( \epsilon \) is a \( (T \cdot \sum_{i=1}^{(a-1)} \) vector of residuals and \( \beta_i = [\beta_i^{(0)} \beta_i^{(1)} \beta_i^{(2)}] \) be corresponding coefficient vector, which is \( ((k+1)x1) \). Define \( Y \) as the vector of price ratios of the weakly separable cluster:
Given this definition of the $Y$ vector the regression is written as:

$$
Y = \begin{bmatrix}
\log \left( \frac{p_1}{p_2} \right) \\
\vdots \\
\log \left( \frac{p_1}{p_a} \right) \\
\log \left( \frac{p_2}{p_3} \right) \\
\vdots \\
\log \left( \frac{p_2}{p_a} \right) \\
\vdots \\
\log \left( \frac{p(a-1)}{p_a} \right)
\end{bmatrix}
$$

(11)

The test for the existence of a aggregate consists of three conditions:

1. $\{(x_i, p_i)\}_{i=1}^T$ satisfies GARP and $U(\cdot)$ exists.

2. $\{(x_i^{(1)}, p_i^{(1)})\}_{i=1}^T$ satisfies GARP and $f(\cdot)$ exists.

3. $\beta_1^{(2)} = \beta_2^{(2)} = \cdots = \beta_{\sum_{i=1}^{a-1}(a-1)}^{(2)} = 0$

where the third condition listed is both necessary and sufficient.

This is a broad representation for weak separability testing in any kind of good or service. For monetary assets included in the broad Divisia index, one can consider the partition of the clustered group to then be $X_t^{(1)} = M_t$ and the partition $X_t^{(2)}$ to contain the outside cluster components, which could consist of personal consumption expenditures on durables, nondurables, services, and leisure.

### 2.3 Stochasticity

The previous theory assumes there is no error in measuring the data; that there is no stochasticity that needs to be accounted for. To deal with noisy observations we assume that there exists a true data generating process, $X^*$, and its observation $X$ includes some independently and identically distributed noise $\psi_{ij}$ with mean zero, variance, $\sigma^2_{\psi_{ij}}$, such that:

$$
x_{ij} = x_{ij}^* + \psi_{ij}
$$

(13)
Even statistically not significant errors in measurement that could lead an erroneous rejection of weak separability, as demonstrated in Barnett and Choi (1989), therefore requiring some adjustment of the before running the Condition 3 regression. de Peretti (2005, 2007) develops such a procedure by finding the minimal level of smoothing needed for the data to satisfy the Condition 1 and Condition 2 GARP tests. If that adjustment is not significantly large then the minimally adjusted data can proceed to the test for Condition 3. However if those adjustments are found to be large, then weak separability must be rejected based on the GARP conditions.

To solve for the minimal adjustment $z_{ij}$, solve a quadratic adjustment program constrained by the GARP conditions:

$$
obj = \min_{z_{ij}} \sum_{i=1}^{T} \sum_{j=1}^{k} (x_{ij} - z_{ij})^2
$$

(14)

for all $i, j$ and $z_{i}^{(1)} = (z_{i1}, z_{i2}, ..., z_{ia})'$ subject to GARP

$$
p_{j} \cdot z_{j} \leq p_{j} \cdot z_{i},
$$

(15)

$$
p_{j}^{(1)} \cdot z_{j}^{(1)} \leq p_{j}^{(1)} \cdot z_{i}^{(1)},
$$

(16)

There are a number of methods to determine if the minimum adjustment is significant as outlined in de Peretti (2005, 2007) and Barnett and de Peretti (2009). Future versions of this paper will use a new econometric technique for which the results are still pending. The preliminary results provided in this version of the paper use the extreme value theory outlined in Barnett and de Peretti (2009).

2.3.1 Extreme Value Theory Analysis

Let the solution matrix to (14) be represented by $Z$ and its matrix of theoretical residuals $\hat{\Omega} = X - \hat{Z}$. These theoretical residuals are then the minimal adjustment needed to satisfy the GARP conditions in (15) and (16). To determine if these adjustments then are significant they should be compared to the true measurement errors, the difference between the observed monetary asset quantities with noise and the true monetary asset noises, $\hat{\Psi} = X - \hat{X}^*$. Those bundles which violate GARP are the ones adjusted, meaning there could be relatively few observations for significance testing. Extreme Value Theory provides a practical solution in testing if the extreme maxima a minima of the theoretical residuals are consistent with measurement error noise. If the extrema are significantly outside the possibilities suggested by $\hat{\Psi}$ then the proposed clustering is rejected.

Relying on the assumption that the theoretical and true residuals are stable in their extremes (they do not explode up or down), define the maximum and minimum for assets $j = 1, ..., k$ and observations $t = 1, ..., T$ as follows. The
vector of estimated theoretical residuals is $(\hat{w}_{1j}, \hat{w}_{2j}), ..., \hat{w}_{Tj}$, the $j$th column of $\hat{\Omega}$ with extreme values:

$$Max_j = \max(\hat{w}_{1j}, \hat{w}_{2j}, ..., \hat{w}_{Tj})$$

$$Min_j = \min(\hat{w}_{1j}, \hat{w}_{2j}, ..., \hat{w}_{Tj})$$

For $(\hat{\psi}_{1,j}, \hat{\psi}_{2,j}, ..., \hat{\psi}_{T,j})$ from $\hat{\Psi}$ the extreme values are:

$$Max_j = \max(\hat{\psi}_{1,j}, \hat{\psi}_{2,j}, ..., \hat{\psi}_{T,j})$$

$$Min_j = -\max(\hat{\psi}_{1,j}, \hat{\psi}_{2,j}, ..., \hat{\psi}_{T,j})$$

If these extreme values are indeed stable then the Fisher Tippet Theorem, acting as a Central Limit Theorem for extreme values, can be applied.

**Fisher-Tippet Theorem 1.** For $\Psi_{i,j} = (\psi_{1,j}, \psi_{2,j}, ..., \psi_{T,j})$ an independently and identically distributed sequence, and existing norming constants $a_{i,j} \in R$ and $b_{i,j} > 0$ and a nondegenerate distribution function $G$ such that

$$(Max_j - a_j)b_j^{-1} \to G$$

then one of three laws can be applied to $G$:

*The Frechet type, for $\alpha_j > 0$:

$$G_F(x) = \begin{cases} 0 & x \leq 0 \\ e^{-x^{-\alpha_j}} & x > 0 \end{cases}$$

*The Weibull type for $\alpha_j < 0$:

$$G_W(x) = \begin{cases} e^{-(-x^{-\alpha_j})} & x \leq 0 \\ 1 & x > 0 \end{cases}$$

*The Gumbel type, for $\alpha_j = 0$ and $x \in R$:

$$G_G(x) = e^{[-e^{-e^{-x}}]}$$

Where the parameter $\alpha_j$ determines the shape of the distribution.

Given a prior amount of information about the true distribution of errors, the above theorem can be used to determine the law of extremes; Frechet ($F$), Weibull ($W$), or Gumbel ($G$). We assume Guassian errors, and therefore use the Gumbel law. By this assumption the tail areas for significance test according to:

$$1 - e^{-\psi_i,\text{max}}$$

for the $p$-value of the maximum and

$$1 - e^{-\psi_i,\text{min}}$$

for the $p$-value of the minimum.
for the p-value of the minimum, such that for location parameters $a_i$ and scale parameters $b_i$

$$y_{i,max} = (\text{Max}_i - a_{i,max})b_{i,max}^{-1}$$

(24)

and

$$y_{i,min} = (\text{Min}_i - a_{i,min})b_{i,min}^{-1}.$$  

(25)

To determine the needed scale and location parameters of the true residual distribution $\Psi_i$, we need an estimation of that distribution. Following one method (of two) proposed in Barnett and de Peretti (2009) uses the initial assumption that the true levels of the assets are linearly related to the observed levels of the assets through additive noise and measurement error. Rewrite this linear equation as a state space model for the unobserved levels $x^*_it$ and uncorrelated errors $\psi_{it}$ and $\xi_{it}$:

$$x_{it} = x^*_{it} + \psi_{it}$$

(26)

$$x^*_{i,(t+1)} = Fx^*_{it} + c_i + \xi_{it}$$

(27)

The hyperparameters for the model are the variances $\sigma^2_\psi$, $\sigma^2_\xi$, and $F$. If a linear trend model is assume then $F = 0$ and $c_i = 0$. From the assumed Gaussian distribution of the residuals and a maximum likelihood estimator $\hat{\sigma}^2_\psi$, of $\psi_{it}$, then the theoretical residuals can be estimated from the minimum adjustment $\hat{\Omega}$, the scale and location parameters, and $\hat{\sigma}^2_\psi$. From the assumption of the Gumbel law and following a procedures outlined in Guegan (2003), the maximum of a series of $T$ draws have a location and scale parameters defined by:

$$a_i = \left(2ln(T)\right)^{\frac{1}{2}} - \left[\frac{ln(ln(T)) + ln(4\pi)}{2(2ln(T))^{\frac{1}{2}}}\right]$$

(28)

$$b_i = \left(2ln(T)\right)^{-\frac{1}{2}}$$

(29)

By the assumption of the stability of the extremes, the location and scale parameters are identical: $a_{i,max} = a_{i,min} = a_i$ and $b_{i,max} = b_{i,min} = b_i$. The estimation for the maximum and minimum are then:

$$\text{Max}_j = \text{Max}_R_j = \max(\hat{\omega}_{1j}\hat{\sigma}_{\psi_j}^{-1}, \hat{\omega}_{2j}\hat{\sigma}_{\psi_j}^{-1}, ..., \hat{\omega}_{Tj}\hat{\sigma}_{\psi_j}^{-1})$$

(30)

$$\text{Min}_j = \text{Min}_R_j = \min(\hat{\omega}_{1j}\hat{\sigma}_{\psi_j}^{-1}, \hat{\omega}_{2j}\hat{\sigma}_{\psi_j}^{-1}, ..., \hat{\omega}_{Tj}\hat{\sigma}_{\psi_j}^{-1})$$

(31)

The above approach provides the methods for testing the first two conditions of the GARP test, to see if the observed data satisfies GARP without significant error caused by noise. Running a Kalman filter on the series of quantity levels provides an estimation of variance for the true residuals and a smoothed level series to use as an instrument for the Condition 3 regression test of the independence of inside cluster levels to outside cluster levels. Once the GARP test is
complete, the remaining procedure is to run the estimation regression using the price ratios and smoothed levels and determine if the coefficient is significantly different from zero.

3 Data

3.1 Broad Divisia Monetary Aggregates

The user cost and quantity data for Divisia monetary components are taken from the data set provided on the Advances in Monetary and Financial Measures website, through the CFS. These data were collected from a variety of accessible resources described in detail in Barnett, Mattson, Liu, and van den Noort (2013). The chosen benchmark rate is the See Table 1 for an organizational chart of the included components.

Seasonal adjustments and splices were necessary in the construction of long term Divisia data. Some components entered into the survey at later times, and others left the survey; for example money market demand accounts became part of the survey in the early 1980s, and were then placed into the an overall measure of savings deposits and MMDA accounts in the early 1990s. All seasonal adjustments used the X-12 ARIMA procedure provided by the Census Bureau and the splicing methodologies are described in Barnett, Mattson, Liu, and van den Noort (2012).

For this paper currency and travelers checks were combined into one component "Currency and Travelers Checks" due to the consistent and precipitous drop in the use of Travelers Checks. Most recently there are fourteen components included in the broadest possible measure, Divisia M4 (DM4), which includes those components measured up to M2 by the Federal Reserve Board and those components discontinued by the Federal Reserve Board that were included in M3, with an additional inclusion of short term treasury debt and commercial paper to make Divisia M4.

For the preliminary results, since the data has undergone several survey changes, the test was split up into four sub categories of time based on the entrance and exit of key monetary asset components. The first sub period tested is 1974:08 to 1982:10, which includes the beginning of the inclusion of retail and institutional money market funds in the Divisia M2 and broader aggregates. The second test time period begins in 1982:12 with the entrance of money market demand accounts and interest bearing checking accounts, and continues until 1991:08. In 1991:09, the beginning of the third period the money market demand accounts are folded into the measures of savings accounts. The final and fourth period beginning in 2006:07 is the month that M3 data is no longer available from the Fed an alternative sources had to be utilized for overnight repurchase agreements, commercial paper, and large denomination time deposits.
3.2 Durables, Nondurables, Services, and Leisure

In the preliminary results of this paper, the monetary components of Divisia M4 and narrower aggregates were first tested against Personal Consumption Expenditures, including durables, nondurables, and services, and used the corresponding deflator as the price. The results from this preliminary test will be reported, however we are currently expanding the tests to specifically test services, durables, nondurables, and leisure to ascertain their separability from monetary assets.

The personal consumption expenditure data and its deflator are taken from the St. Louis Federal Reserve Bank’s Federal Reserve Economic Data tool. Data and estimation for the current and on-going tests determined the prices and levels of other asset groupings based on the work of Diewert (1974), Barnett (1978), Drake (1994), and Patterson (1991).

The levels and prices of leisure are calculated using the average hours of production and nonsupervisory workers and the shadow cost of leisure as derived in Barnett (1978).

Nondurable prices and levels were determined according to Patterson (1991) and the work of Diewert (1974) in determining the rental price. The rental price for a durable good \( p_{ij} \) for some asset \( j \) at time \( i \) is determined by the rate of depreciation \( \delta_i \) of that asset, which we assume to be 10%, the benchmark rate of return given up to hold that durable, \( r^B_i \), the inflation rate \( \Pi_{ij} \), ad the prevailing observed price \( j \) at \( i \) \( p^*_ij \):

\[
p_{ij} = \frac{p^*_ij [r^B_i + \delta_i (1 + \Pi_{ij}) - \Pi_{ij}]}{1 + r^B_i} \quad (32)
\]

The durable levels are calculated as stocks based on the initial purchase at the initial time period such that:

\[
S_i = I_i + (1 - \delta_i)S_{i-1} \quad (33)
\]

for \( S_i \) the current period stock, \( I_i \) the current period expenditure on stock, \( S_{i-1} \) the previous period stock, and \( S_1 = I_1 \).

Nondurable goods and services provide more straightforward pricing and levels. Depreciation is considered to be 100% and the user cost is the market price paid for the nondurable. The level is the stock purchased in that period as no previous period nondurable goods survive into the next period. Similarly services are priced with their expenditure and price in time period \( i \).

Stock of leisure is nonmarket time, following the procedure outlined in Swofford and Whitney (1987). Nonmarket time is 98 hours less the average hours worked by nonsupervisory employees in the United States, multiplied by four to get monthly observations. Leisure requires the determination of the shadow cost of leisure as outlined in Barnett 1979 Chapter 2. The going wage rate observed, in this case the average hourly earnings of nonsupervisory employees does not take into account the affect on the price of leisure of the employment rate in the economy. For full employment, the shadow price of leisure would
equal the wage rate, however in times of higher unemployment the price of leisure would not be the same as the wage rate. For some observed wage rate $w$ and level of employment $E$ the shadow cost of leisure which takes into account the variations in wages as well as employment is defined as:

$$\hat{w}_i = w_i E_i^{\alpha}$$  \hspace{1cm} (34)

In this equation $\alpha$ is a parameter defining the effect of employment on the shadow cost of leisure and is estimated in Barnett (1979) to be $\alpha = 2.3$; a measure we will use as well.

These considerations for durables, nondurables, services, and leisure have not yet been incorporated into our results for the GARP test. We are still able to present our preliminary results of the GARP test showing separability of monetary components from the broad Personal Consumption Expenditures aggregate.

4 Results

First the broadest possible monetary aggregate is tested for weak separability from personal consumption expenditures. For the first period starting in 1974 and continuing to 1982, the maximum adjustment was found to still be within the non-significant range of the extreme value behavior, supporting the presence of GARP, the minimum value has a sever violation in the last months of 1982. The violation coincides with the emergence of money market demand accounts and the changing in survey data methodology for interest bearing checking accounts; “other checkable deposits”. Interestingly the violation also occurs during the Savings and Loan Crisis of the early 1980s. If the time period is shortened (as is common the parametric literature for weak separability test) to not include these violations, all other violations are found to be non significant and the GARP conditions would be satisfied for this period. All other components of the Divisia monetary aggregates support non-rejection of GARP for DM4 and DM4-. The Condition 3 weak separability test supports DM4 and DM4-, with minimum tail areas that do not support rejection at a 5% level.

The end of 1982 to mid 1991 time period proved the most problematic for satisfying the Condition 3 independence test. DM4 performs better with a lowest tail area of 0.0231 while the other broad aggregates have a minimum tail area that falls below 0.000. DM4 performed better with the GARP conditions, with no evidence for rejecting Conditions 1 and 2, even in the problematic interest bearing checking accounts. DM3 and DM4- again do not pass the GARP conditions due to violations in interest bearing checking accounts related to late 1982 and early 1983.

The time period from 1991 to 2006 is the longest with $T = 175$ observations. All broad aggregates satisfy the GARP conditions, with no errors outside the significance range. The independence of th price ratios however lends support only to Divisia M4 with a smallest p-value of 0.0131. DM4- and DM3 in this su period have smallest p-values of 0.0060 and 0.000.
The last time period again supports DM4 with the GARP conditions, but not DM3. The price ratio independence test supports DM4 as well with a lowest p-value of 0.0678.

Alternative narrower aggregates were tried as well including the narrow aggregates provided by the Federal Reserve, M2 ALL and MZM, paired with their user cost prices. As shown in Table 1 the narrower aggregates failed to satisfy GARP, or in some cases passed the Condition 1 and 2 tests only with tail areas less than 5%. Alternative measures of DM3 that exclude retail or institutional money market funds passed the GARP conditions but failed the weak separability test in all time periods.

5 Conclusion and Extensions

The preliminary results indicate support for use of the broad Divisia monetary aggregates over narrower forms that do not include assets financial market assets that provide liquidity such as overnight repurchases, commercial paper, large time deposits, and short term treasury bills. The test used in this paper extends a new branch in the non-parametric weak separability literature beginning in Varian (1982,1983) and more recently extending through the work of de Peretti (2005, 2007) and Barnett and de Peretti (2009). Stochastic factors are accounted for a quadratic programming formula and Kalman filter to determine the significance of GARP violations, easing the problem pointed out originally in Barnett and Choi (1998). The independence test of price ratios and levels provides a necessary and sufficient condition for confirmation of weak separability, an improvement on the Afriat inequality work in Swofford and Whitney (1987) and Patterson (1991).

Coupled with the extension of the non parametric clustering literature is the use of new broad Divisia monetary aggregates which provide a better view of the overall economy and monetary policy, especially given recent experience at the zero lower bound when interest rates are not providing as much information regarding monetary services as before. The non-parametric clustering can identify which aggregates are to be used and which components should be included within those aggregates, providing a data driven definition of money that allows for innovations in financial vehicles and movements related to recessions and expansions.

The clustering however is incomplete in that we have yet to expand the test using sub-aggregates of durable and non-durable goods, leisure, and services, as used in Patterson (1991) and Swofford and Whitney (1987). While the use of personal consumption expenditures is in line with literature on Divisia aggregates for the European Union, specifically Binner, Bissoondeel, Elger, Jones, and Mullineux (2008), the inclusion of short term treasury debt and broader financial vehicles brings up the question of whether these goods fit with another non-monetary category instead.

There are several natural extensions to this literature including risk adjustment of user costs, analysis of causal relations, and inflation and exchange rate
forecasting. The user costs in this project do not account for the presence of risk in the market of the components. Barnett and Wu (2005) develop a process that uses a CAPM model to estimate the risk adjusted user cost can be readily applied to the weak separability test by simply replacing the risk adjusted user costs with the non-risk user costs tested here. It should be recognized that the risk environment in the US has fluctuated over a long period of time, and more recently in a very short period of time. A working paper by Mattson (2013) incorporates this risk adjustment, and so far has found that adjusting for risk improves the performance of DM4 and DM4- in the GARP tests. Given the determination of admissibility, the aggregates can be used in exchange rate forecasting as in Barnett and Kwag (2006) and Biissoondeeal, Karoglou, and Gazely (2010) who find use of Divisia aggregates in exchange rate forecasting models to outperform simple sum measures. Inflation forecasting has not, to the authors knowledge, been done with US broad Divisia data provided by the CFS, but has been done in Alkhareif and Barnett (...) for the Gulf Coast Countries.

6 Tables and Figures

### Table 1: Admissibility Test Results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>G**/WS*</td>
<td>G**</td>
<td>G#</td>
<td>G*</td>
<td>G*</td>
</tr>
<tr>
<td>Divisia M4 Less RMF</td>
<td>G**</td>
<td>G#</td>
<td>G#</td>
<td>X</td>
</tr>
<tr>
<td>Divisia M4 Less MMF</td>
<td>G**</td>
<td>G#</td>
<td>G#</td>
<td>X</td>
</tr>
<tr>
<td>Divisia M4+</td>
<td>G**</td>
<td>X</td>
<td>G#</td>
<td>G#</td>
</tr>
<tr>
<td>Divisia M3</td>
<td>X</td>
<td>X</td>
<td>G**</td>
<td>G**</td>
</tr>
<tr>
<td>Divisia M3 Less RMF</td>
<td>G**</td>
<td>G#</td>
<td>G#</td>
<td>G#</td>
</tr>
<tr>
<td>Divisia M3 Less MMF</td>
<td>G**</td>
<td>G#</td>
<td>G#</td>
<td>G#</td>
</tr>
<tr>
<td>Divisia M2 ALL</td>
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<td>X</td>
<td>X</td>
<td>G**</td>
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<tr>
<td>Divisia M2M</td>
<td>G**</td>
<td>X</td>
<td>X</td>
<td>G**</td>
</tr>
</tbody>
</table>

_Notes:_ G denotes failure to reject Conditions 1 and 2, but rejection of Condition 3. G/WS denotes failure to reject all three conditions. X denotes a rejection of all conditions.

_Sources_: Author calculations.

* Significant at the 1 percent level
* Significant at the 5 percent level.
### Table 2 GARP 1974 Results

**TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1974:1982, T=100**

<table>
<thead>
<tr>
<th></th>
<th>Divisia M3</th>
<th>Divisia M4-</th>
<th>Divisia M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency and Travelers Checks</td>
<td>.3664 / .2034</td>
<td>.3668 / .2229</td>
<td>.4043 / .3085</td>
</tr>
<tr>
<td>Demand Deposits</td>
<td>.5936 / .7928</td>
<td>.7011 / .6189</td>
<td>.7182 / .6717</td>
</tr>
<tr>
<td>Other Checkable Deposits Comm.</td>
<td>.3387 / .6932</td>
<td>.3557 / .6808</td>
<td>.3516 / .6859</td>
</tr>
<tr>
<td>Other Checkable Deposits Thrift</td>
<td>.0033 / .0921</td>
<td>.0284 / .3362</td>
<td>.0329 / .3460</td>
</tr>
<tr>
<td>Savings Less MMDA Comm.</td>
<td>.7159 / .8148</td>
<td>.7273 / .8160</td>
<td>.7364 / .8151</td>
</tr>
<tr>
<td>Savings Less MMDA Thrift</td>
<td>.6232 / .5609</td>
<td>.4244 / .6702</td>
<td>.4479 / .6754</td>
</tr>
<tr>
<td>Retail Money Market Funds</td>
<td>.8386 / .7888</td>
<td>.6966 / .6666</td>
<td>.6952 / .6385</td>
</tr>
<tr>
<td>Small Time Deposits Comm.</td>
<td>.8464 / .8052</td>
<td>.8472 / .8071</td>
<td>.8466 / .7722</td>
</tr>
<tr>
<td>Small Time Deposits Thrift</td>
<td>.7111 / .6819</td>
<td>.8433 / .7990</td>
<td>.8427 / .7604</td>
</tr>
<tr>
<td>Institutional Money Market Funds</td>
<td>.8647 / .8430</td>
<td>.8046 / .7909</td>
<td>.8031 / .7786</td>
</tr>
<tr>
<td>Large Time Deposits</td>
<td>.7269 / .6818</td>
<td>.8515 / .8161</td>
<td>.8510 / .7853</td>
</tr>
<tr>
<td>Overnight and Term Repos</td>
<td>.7043 / .5824</td>
<td>.8176 / .7835</td>
<td>.8367 / .7985</td>
</tr>
<tr>
<td>Commercial Paper</td>
<td>(.8296 / .8076)</td>
<td>.6576 / .6109</td>
<td>.7070 / .6543</td>
</tr>
<tr>
<td>Short Term Treasury Bills</td>
<td>(.8618 / .8746)</td>
<td>(.8870 / .8870)</td>
<td>.8191 / .8021</td>
</tr>
<tr>
<td>Personal Consumption Expenditures</td>
<td>NA</td>
<td>NA</td>
<td>(8870 / 8870)</td>
</tr>
</tbody>
</table>

*Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Subutility).*

*Source: Author calculations.*

### Table 3 GARP 1982 Results

**TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1982:1991, T=105**

<table>
<thead>
<tr>
<th></th>
<th>Divisia M3</th>
<th>Divisia M4-</th>
<th>Divisia M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency and Travelers Checks</td>
<td>.5722 / .7053</td>
<td>.6251 / .6375</td>
<td>.7183 / .7983</td>
</tr>
<tr>
<td>Demand Deposits</td>
<td>.7831 / .8268</td>
<td>.8010 / .8050</td>
<td>.8310 / .8357</td>
</tr>
<tr>
<td>Other Checkable Deposits Comm.</td>
<td>.2499 / .6234</td>
<td>.3110 / .7612</td>
<td>.7036 / .7359</td>
</tr>
<tr>
<td>Other Checkable Deposits Thrift</td>
<td>.0063 / .2387</td>
<td>.0129 / .5197</td>
<td>.3736 / .4750</td>
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<tr>
<td>MMDA Commercial</td>
<td>.8388 / .8093</td>
<td>.8196 / .8499</td>
<td>.8405 / .8566</td>
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<tr>
<td>MMDA Thrift</td>
<td>.8183 / .7741</td>
<td>.7897 / .8340</td>
<td>.8209 / .8434</td>
</tr>
<tr>
<td>Savings Less MMDA Comm.</td>
<td>.8558 / .8209</td>
<td>.8477 / .8651</td>
<td>.8558 / .8606</td>
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<tr>
<td>Savings Less MMDA Thrift</td>
<td>.7946 / .6477</td>
<td>.7673 / .8221</td>
<td>.7950 / .8381</td>
</tr>
<tr>
<td>Retail Money Market Funds</td>
<td>.7057 / .7483</td>
<td>.7981 / .8466</td>
<td>.7985 / .7560</td>
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<tr>
<td>Small Time Deposits Comm.</td>
<td>.8049 / .8270</td>
<td>.8414 / .7609</td>
<td>.8423 / .8302</td>
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<tr>
<td>Small Time Deposits Thrift</td>
<td>.7965 / .8210</td>
<td>.8369 / .7478</td>
<td>.8324 / .8245</td>
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<tr>
<td>Institutional Money Market Funds</td>
<td>.8079 / .8260</td>
<td>.8498 / .8677</td>
<td>.8493 / .8293</td>
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<td>Large Time Deposits</td>
<td>.8142 / .8337</td>
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<td>.8217 / .8365</td>
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<td>.8228 / .7754</td>
<td>.8675 / .8096</td>
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<tr>
<td>Commercial Paper</td>
<td>(.8835 / .8757)</td>
<td>.8225 / .7718</td>
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<td>Short Term Treasury Bills</td>
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<td>(.8875 / .8875)</td>
<td>.8783 / .8603</td>
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<td>Personal Consumption Expenditures</td>
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<td>(8875 / 8875)</td>
</tr>
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*Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Subutility).*

*Source: Author calculations.*
**Table 4 GARP 1991 Results**

<table>
<thead>
<tr>
<th>Tail Area Results of the GARP Test on Broad CFS Clusters 1991:2006, T=175</th>
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<tr>
<td>Demand Deposits</td>
</tr>
<tr>
<td>Other Checkable Deposits Comm.</td>
</tr>
<tr>
<td>Other Checkable Deposits Thrift</td>
</tr>
<tr>
<td>Savings and MMDA Comm.</td>
</tr>
<tr>
<td>Savings and MMDA Thrift</td>
</tr>
<tr>
<td>Retail Money Market Funds</td>
</tr>
<tr>
<td>Small Time Deposits Comm.</td>
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<tr>
<td>Small Time Deposits Thrift</td>
</tr>
<tr>
<td>Institutional Money Market Funds</td>
</tr>
<tr>
<td>Large Time Deposits</td>
</tr>
<tr>
<td>Overnight and Term Repos</td>
</tr>
<tr>
<td>Commercial Paper</td>
</tr>
<tr>
<td>Short Term Treasury Bills</td>
</tr>
<tr>
<td>Personal Consumption Expenditures</td>
</tr>
</tbody>
</table>

*Notes:* “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Subutility).

*Source:* Author calculations.

**Table 5 GARP 2006 Results**

<table>
<thead>
<tr>
<th>Tail Area Results of the GARP Test on Broad CFS Clusters 2006:2012, T=75</th>
</tr>
</thead>
<tbody>
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<td>Currency and Travelers Checks</td>
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<tr>
<td>Demand Deposits</td>
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<tr>
<td>Other Checkable Deposits Comm.</td>
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<tr>
<td>Other Checkable Deposits Thrift</td>
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<tr>
<td>Savings and MMDA Comm.</td>
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<tr>
<td>Savings and MMDA Thrift</td>
</tr>
<tr>
<td>Retail Money Market Funds</td>
</tr>
<tr>
<td>Small Time Deposits Comm.</td>
</tr>
<tr>
<td>Small Time Deposits Thrift</td>
</tr>
<tr>
<td>Institutional Money Market Funds</td>
</tr>
<tr>
<td>Large Time Deposits</td>
</tr>
<tr>
<td>Overnight and Term Repos</td>
</tr>
<tr>
<td>Commercial Paper</td>
</tr>
<tr>
<td>Short Term Treasury Bills</td>
</tr>
<tr>
<td>Personal Consumption Expenditures</td>
</tr>
</tbody>
</table>

*Notes:* “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Subutility).

*Source:* Author calculations.
### Table 6 Weak Separability Test

**Tail Area Results of Condition 3 Weakly Separable Test**

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Time Period</th>
<th>Sample Size</th>
<th>Min Tail Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM4</td>
<td>1971:1982</td>
<td>100</td>
<td>0.1164</td>
</tr>
<tr>
<td></td>
<td>1982:1991</td>
<td>105</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td>1991:2006</td>
<td>175</td>
<td>0.0131</td>
</tr>
<tr>
<td></td>
<td>2006:2012</td>
<td>75</td>
<td>0.0678</td>
</tr>
<tr>
<td>DM4-</td>
<td>1971:1982</td>
<td>100</td>
<td>0.0056</td>
</tr>
<tr>
<td></td>
<td>1982:1991</td>
<td>105</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>1991:2006</td>
<td>175</td>
<td>0.0060</td>
</tr>
<tr>
<td></td>
<td>2006:2012</td>
<td>75</td>
<td>0.0041</td>
</tr>
<tr>
<td>DM4 Less RMF</td>
<td>1971:1982</td>
<td>100</td>
<td>0.0027</td>
</tr>
<tr>
<td></td>
<td>1982:1991</td>
<td>105</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>1991:2006</td>
<td>175</td>
<td>0.0013</td>
</tr>
<tr>
<td></td>
<td>2006:2012</td>
<td>75</td>
<td>0.0401</td>
</tr>
</tbody>
</table>

*Notes:* The smallest minimum tail area of the price ratios is reported. Tail areas below .01 are immediately rejected while those between .01 and .10 are reported.

*Source:* Author calculations.
References


