Inside Money, Business Cycle, and Bank Capital Requirements

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April 13, 2018

Abstract

A search theoretical model is constructed to study bank capital requirements in a respect of inside money. In the model bank liabilities, backed by bank assets, are useful for exchange, while bank capital is not. When the supply of bank liabilities is not sufficiently large for the trading demand, banks do not issue bank capital in competitive equilibrium. This equilibrium allocation can be suboptimal when the bank assets are exposed to the aggregate risk. Specifically, a pecuniary externality is generated because banks do not internalize the impact of issuing inside money on the asset prices in general equilibrium. Imposing a pro-cyclical capital requirement can improve the welfare by raising the price of bank assets in both states. **Key Words**: constrained inefficiency, pecuniary externality, limited commitment **JEL Codes**: E42, E58

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[†]I am greatly indebted to Stephen Williamson for his continuous support and guidance. I am thankful to John Conlon for his dedicated advice on this paper. This paper has also benefited from the comments of Gaetano Antinolfi, Costas Azariadis and participants at Board of Governors of the Federal Reserve System, Korean Development Institute, The University of Mississippi, Washington University in St. Louis, and 2015 Mid-West Macro Conference at Purdue University. All errors are mine.

1 Introduction

Why do we need to impose capital requirements to banks? If needed, should it be pro-cyclical or counter-cyclical? A conventional rationale for bank capital requirements is based on deposit insurance: Banks tend to take too much risk under this safety net, so bank capital requirements are needed to correct the moral hazard problem created by deposit insurance. Alternatively, systemic risk also justifies capital requirements on banks because a failure of one bank may lead to a chain reaction in which many other banks can go bankrupt. In this case capital can function as a buffer to prevent the default of the banks. Specifically, pro-cyclical capital requirements are proposed in practice since it enforces banks to accumulate bank capital in credit boom to mitigate credit crunch in the following recession.[1] These two rationales for bank capital requirements focus mainly on the financial intermediation function of the banks, while banks play a variety of roles in an economy. Thus, it would be worthwhile to evaluate bank capital requirements in the other perspectives of the banks.

One primary function of banks is to provide methods of payment to facilitate transactions. For example, bank liabilities such as deposit claims and bank notes had been used either as a medium of exchange in retail markets or as collateral for secured credit in the interbank markets.[2] If bank liabilities are useful for transactions while bank capital is not or less useful, then bank capital requirements can be used to manage the supply of liquidity in an economy.[3] In this paper I study a new role of capital requirements by focusing on this liquidity provision function of the banks where banks issue inside money backed by their asset portfolio.

In order to explore this issue I develop a search-theoretical model a la Lagos and Wright (2005) with a banking arrangement shown in Williamson (2012). This micro-founded model has an advantage of incorporating informational frictions such as limited commitment and imperfect memory in a simple way and is highly tractable with an array of assets and banking contracts. This framework is also suitable for welfare analysis in that the cost for holding assets is determined endogenously in the model. The main features of the model are as follows. Agents can produce consumption goods with an elastic labor supply, but cannot consume their own output. Agents need a medium of exchange to trade each other under limited commitment and lack of memory, but only bank liabilities are accepted for

[1]BCBS (2010a,b) introduced a new pro-cyclical component which requires additional bank capital from 0 percent to 2.5 percent at regulators' discretion.

[2]Bank notes were used as a medium of exchange during the Free banking era and the National banking era in the United States. At that time bank equity (or capital) was not actively used in transactions.

[3]Gorton and Pennacchi (1990) explain the liquidity difference between debt and equity with asymmetric information. Diamond and Rajan (2000) show that holding bank capital can rather increase the liquidity of bank liabilities with a demandable deposit contract. Berger and Bouwman (2009) find out that higher capital ratios create less liquidity for small banks whereas more liquidity for large banks by using data on U.S. banks in 1993-2003.

transactions while bank capital is not. Given limited commitment, the issued bank liabilities and capital must be secured by asset holdings of the banks.[4] So if the supply of the underlying assets is insufficient to support the demand for transactions, the consumption level of agents can be restricted. The assets in this economy are exposed to a non-diversifiable risk in a form of a random dividend, high or low. Everyone knows the realization of the dividend before the consumption period. So there is no asymmetric information in transactions and the consumption level of the agents fluctuates by the realization of the dividend.

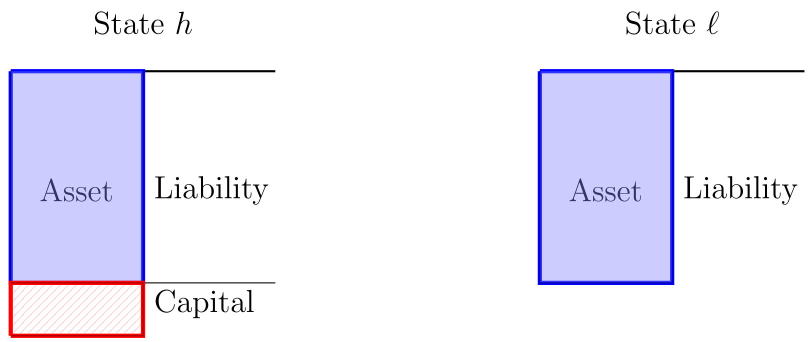

Figure 1. Bank balance sheet with a pro-cyclical capital requirement

Raising bank capital naturally reduces the proportion of bank liabilities, so that the supply of liquidity in an economy decreases. Specifically, state-contingent bank capital can reduce the proportion of underlying assets that is used for exchange by states.[5] For example, consider a state-contingent bank capital, which promises a proportion of bank assets in state h while nothing in state ℓ. If banks sell this type of bank capital, in case of state h the rest of bank assets, the blue shaded rectangle in Figure 1, can support transactions, but the red slashed rectangle cannot. On the other hand, in case of state ℓ the whole bank assets can be used for exchange. If the supply of the assets is not sufficiently large to support transactions in both states, it is not profitable for banks to issue a strictly positive bank capital since it is useless for trade. Given the scarcity of assets banks provide only liabilities without bank capital in competitive equilibrium, so that the consumption level of the agents fluctuates by the realization of the state.

This competitive equilibrium allocation can be constrained suboptimal. Given a stable transactions demand across states, requiring a bank capital, which provides a proportion of

[4] Given the limited commitment, when banks run away their assets are seized and transferred to the holders of bank liabilities and capital.

[5] We can interpret that bank capital reduces the pledgeability of bank assets since the proportion of assets, which can be seized when a bank defaults, decreases. Thus, if assets can be traded directly without banking contracts, hair-cut and/or loan-to-value ratio regulations can make the same effect by reducing the pledgeability of the assets.

assets in state h while nothing in state ℓ, can improve welfare by raising the asset prices in both states. Requiring a proportion of assets in state h lowers the supply of liquidity for transactions and the consumption level in state h. However, restricting the liquidity supply in state h can raise the liquidity premium in state h, so that the prices of the bank assets in both states rise in general equilibrium. The consumption level in state ℓ would increase as the asset prices rise, because the whole bank asset portfolio is used for transactions in state ℓ. Therefore, there is a trade-off between a direct effect of capital requirements, which reduces the consumption in state h by restricting liquidity, and an indirect effect of capital requirements, which raises the consumptions in both states by relaxing collateral constraints with the higher asset prices. Thus, provided that agents are risk-averse, bank capital requirements can improve welfare by smoothing the marginal utilities between the states h and ℓ.

This constrained inefficiency of competitive equilibrium is associated with a pecuniary externality of the banking sector. The asset prices are determined in the competitive asset market where the banks purchase assets and issue liabilities and capital given the asset prices. Thus, individual banks cannot internalize the impact of issuing a positive bank capital on the price of the asset in general equilibrium.[6] In other words, although issuing a strictly positive bank capital is welfare-enhancing for all of the banks, the banks will not issue bank capital in equilibrium since it is not profitable for them given the prices. This pecuniary externality result can provide another rationale for bank capital requirements in a perspective of issuing liquidity.

This finding provides a new insight for business cycle stabilization. In the paper requiring higher capital in state h transfers a purchasing power from state h to state ℓ by raising the prices of the assets in both states.[7] Therefore, when storing or transferring consumption goods from one state to the other is limited, bank capital requirements can be useful to transfer a purchasing power by adjusting the asset prices. The result of this paper also suggests a pro-cyclical capital requirement to stabilize business cycles, but the main mechanism is different from the literature on capital buffer which requires a real transfer from expansion periods to recession periods.

Finally, I extend the model by introducing a nominal debt issued by the government, called as money, in order to compare the effect of capital requirements with the one of monetary policy. The government can adjust the price of money by providing lump-sum transfers or collecting taxes, so that a state-contingent monetary policy can transfer a purchasing

[6] In this respect we need to study further on the market structure of banks to correct this pecuniary externality. For example, in the case of monopoly, the monopoly bank would internalize the effect on the asset price to maximize its profit.

[7] Note that the main result of this paper is robust in a deterministic case without uncertainty, since the asset price in one state reflects the asset prices in the other states as long as there is a positive transition probability.

power across states without incurring the cost of requiring bank capital. However, the effect of monetary policy is limited by the outstanding money balance while bank capital requirements can be applied to the whole asset portfolio of banks including money and other private assets.

Related Literature

There is a vast theoretical literature on bank capital regulations.[8] One strand of this literature focuses on the risk-shifting behavior of banks and the cost of default.[9] Recent theoretical papers on this strand, including Malherbe (2015), Boissay and Collard (2016), and Magill et al. (2016), show how bank capital requirements can correct the (pecuniary) externality associated with the default risk.[10]

The other theoretical strand of the literature examines the externality associated with fire sales and systemic risk.[11] Recent papers including Lorenzoni (2008), Jeanne and Korinek (2013), Stein (2012), Goodhart et al. (2012), Farhi and Werning (2016), and Gale and Gottardi (2017) pay attention to the macroeconomic effect of the systemic risk and policy responses.[12] Specifically, Lorenzoni (2008) studies a fire sales externality associated with excessive borrowing and shows how a capital requirement can correct it. Farhi and Werning (2016) consider macro-prudential policy to internalize systemic risk in the incomplete market.

There are few papers that study bank capital requirements in a quantitative way. Nguyen (2014) develops a model in which the government bailout can induce the over-leverage of the banks and shows that the optimal capital requirement is greater than the current level in the United States. Begenau (2016) quantifies a model that bank capital requirements can rather increase bank lending unlike the previous literature, when bank liabilities are used as a means of payment. She shows that the level of capital requirements in the U.S. is substantially low given the parameters.

[8]See VanHoose (2007) for a review of the theoretical literature on bank capital regulations.

[9]As classic papers, Kareken and Wallace (1978), Kim and Santomero (1988), and Furlong and Keeley (1989) show that deposit insurance can create moral hazard of banks. Recently, Dewatripont and Tirole (2012) and Boyd and Hakenes (2014) focus on the managerial looting incentive rather than risk-taking behavior.

[10]In Malherbe (2015) when bank lending affects the expected default costs of other banks, individual banks do not internalize the social cost of default. Boissay and Collard (2016) study the relationship between market liquidity conditions and the moral hazard of banks: Fewer liquid assets could lead banks issue more deposit, so that the moral hazard incentive of the banks could increase. Magill et al. (2016) also show a path that the shortage of safe assets can increase the default risk of banks.

[11]For the existence of systemic risk, Allen and Gale (2007) study an environment where credit risk is insufficiently transferred to the insurance institutions to eliminate systemic risk.

[12]In order to correct the externality related to systemic risk, Jeanne and Korinek (2013) propose a pigouvian tax on short-term liabilities while Stein (2012) suggests trading permits for banks to issue short-term debt. Goodhart et al. (2012) also explore various types of financial regulations to reduce fire sales. Gale and Gottardi (2017) claim that firms or diversified banks must hold sufficient equity to absorb a negative shock.

This paper complements the recent literature that studies the bank liability channel in macroeconomic models. Quadrini (2017) shows that when banks can issue liabilities which are useful for insurance and economic activities, a liquidity dry-up is feasible by self-fulfilling expectations. Bank liabilities are also used as inside money in Li (2017) and Benigno and Robatto (2017). Li (2017) develops an amplification mechanism from the interaction between bank's money creation and firm's investment. When a default of bank loan reduces the supply of bank liability, the firm's investment can decline because of liquidity contraction. Benigno and Robatto (2017) studies a pecuniary externality associated with the supply of pseudo-safe debt. Given a cost for issuing equity, banks do not internalize the effect of risky debt issuance when a default can reduce the aggregate liquidity. My paper is close to Benigno and Robatto (2017) in a respect that banks do not internalize the effect of issuing inside money. However, my paper focus more on the relationship between the prices of bank assets and the liquidity of bank liabilities under limited commitment.

In my paper inside money of banks is supported by pledgeable assets under limited commitment, so this paper is also related to the broad literature on secured credit.[13] The seminal paper in this literature is Kiyotaki and Moore (1997) which explains large volatilities in the asset prices with an amplification mechanism. In my paper there is no amplification effect because the quasi-linearity of the model shut down the income effect. The path from the current shock to the future credit conditions is cut off, but the capital requirements for the future periods are still effective in the current asset price.[14] Thus, the constrained inefficiency in the model is derived without relying on the amplification effect.

This paper finds a pecuniary externality in a cyclical economy with one good and one asset under limited commitment. This pecuniary externality is related to the generic results of Stiglitz (1982), Geanakoplos and Polemarchakis (1986), and Kehoe and Levine (1993) in which the competitive equilibrium is constrained sub-optimal with incomplete market.[15] In Kehoe and Levine (1993) the first welfare theorem holds for one good, but it can fail for two or more goods with endogenous borrowing constraints. Similarly, the constrained inefficiency in this paper is generated because the relative asset price across the states is misaligned with the marginal rate of substitution under limited commitment. Thus, there is a possibility that an additional constraint such as capital requirements can improve the welfare of the equilibrium allocation.

Finally, this paper relies on the recent burgeoning literature of the shortage of safe as-

[13]Kiyotaki and Moore (2002) concentrate on limited commitment to model inside money, and Andolfatto and Nosal (2009) show that banks can issue inside money based on their monitoring ability under limited commitment.

[14]In the search theoretical framework, Venkateswaran and Wright (2014) study how the pledgeability of assets can affect the liquidity premium in the asset prices.

[15]While these papers focus on the market incompleteness, my paper concentrates more on the limited commitment of banks to show the pecuniary externality.

sets.[16] In particular, Krishnamurthy and Vissing-Jorgensen (2012) find empirically that the yield on U.S. treasuries over 1926-2008 has a liquidity premium for moneyness and safety. Caballero et al. (2016) support the idea that low real interest rates on government debt can be explained by the shortage of safe assets in both empirical and theoretical ways.[17] In my paper the aggregate shortage of safe assets is the key feature to determine bank liability-capital structure as shown in Williamson (2016), because the total supply of assets changes the cost for issuing bank capital. This model structure is different from the previous papers, in which bank capital itself is scarce. In Gertler and Kiyotaki (2015), banks need to accumulate costly bank capital to raise funds from depositors, but the supply of bank capital is insufficient. In my model the bank liability-capital structure influences the prices of bank assets and simultaneously the prices of bank assets determines the total supply of assets which is available for issuing bank liabilities.

2 Model

The model structure is based on Rocheteau and Wright (2005) in which ex ante heterogeneous agents trade in bilateral decentralized meetings and rebalance their asset portfolio in competitive and centralized asset markets. Time $t = 0, 1, 2, \ldots$ is discrete and the horizon is infinite. Each period is divided into two sub-periods - the centralized market (CM) followed by the decentralized market (DM). There is a continuum of buyers, sellers and bankers, each with unit mass. All the agents can consume and produce in the CM. But in the DM buyers can consume, but cannot produce while sellers can produce, but cannot consume. One unit of labor input produces one unit of perishable consumption good either in the CM or in the DM.[18]

An individual buyer has preferences,

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

where $H_t \in \mathbb{R}$ is labor supply and sales of the buyer in the CM, $x_t \in \mathbb{R}_+$ is consumption of the buyer in the DM, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u^{'}(0) = \infty, u^{'}(\infty) = 0$, and $-\frac{xu^{''}(x)}{u^{'}(x)} = \gamma < 1$.[19]

[16] For further survey, see Golec and Perotti (2015) and Gorton (2017).

[17] Beyond that, He et al. (2016) study the conditions to be safe assets: For example, large debt size is beneficial to reduce the rollover risk. Andolfatto and Williamson (2015) and Caballero and Farhi (2017) study the policy restrictions generated by the shortage of safe assets.

[18] CM and DM consumption goods are not necessary to be the same.

[19] Constant Relative Risk Aversion(CRRA) utility function is useful because given CRRA utility function, the substitution effect dominates the income effect and the optimal policy is unique. Specifically, given

Each seller has preferences,

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t],$$

where $X_t \in \mathbb{R}$ is consumption of the seller in the *CM*, and $h_t \in \mathbb{R}_+$ is labor supply and sales of the seller in the *DM*. An individual banker has preferences,

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t^b - H_t^b],$$

where $H_t^b \in \mathbb{R}_+$ is labor supply of the banker in the *CM*, and $X_t^b \in \mathbb{R}_+$ is consumption of the banker in the *CM*.[20]

In the *DM* each buyer meets one seller, and vice-versa, bilaterally. The terms of trade are determined by bargaining, with the buyer making a take-it-or-leave-it offer to the seller.[21] There is no record-keeping technology for buyers and sellers, so that they are anonymous. Under limited commitment no one can be forced to work. Thus, recognizable assets are essential for trade in the *DM*, and the trade must be *quid pro quo*. I assume that bankers cannot participate in the *DM* trade, but the banker's information such as names, addresses, and asset holdings, is available to buyers and sellers.[22] Therefore, deposit claims issued by the banker in the *CM* can be recognized by buyers and sellers in the *DM*.[23] Under limited commitment, if the bankers abscond in the next *CM*, the underlying bank assets would be seized and transferred to the holders of the deposit claims.[24] This is the reason why the deposit claims are used as inside money in the *DM*. Thus, bankers cannot participate in the *DM*, but they play a role of providing a medium of exchange in the model.

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the definition of $g(x) = xu^{'}(x)$, an assumption $g^{'}(x) > 0$ is needed to confirm that the substitution effect dominates the income effect. If $g^{'}(x) \leq 0$ then when the demand for the asset increases, the price of the asset can go down. The assumption $g^{''}(x) < 0$ is also needed to avoid local solutions for the optimal policy. If $g^{''}(x) \geq 0$ then the price of the asset can increase more rapidly when the level of consumption decreases, so we cannot confirm that there exists a unique optimal policy. One simple case that satisfies $g^{'}(x) > 0$ and $g^{''}(x) < 0$ is Constant Relative Risk Aversion(CRRA) utility function with $-\frac{xu^{''}(x)}{u^{'}(x)} = \gamma < 1$ because $g^{'}(x) = (1-\gamma)u^{'}(x)$.

[20] We can introduce a banking contract which issues both deposits and capital instead of bankers and let buyers and/or sellers purchase capital. The maximization problem with the banking contract is equivalent to the maximization problem with the bankers. However, we need an additional consideration for determining whether banks maximize the depositor's value or the equity holder's value.

[21] The competitive prices of *DM* good and the asset in the *DM* are not defined explicitly in the model because there is no centralized asset market in the *DM*. However, given the bargaining structure, sellers are willing to provide β unit of the *DM* good in the *DM* for one unit of the *CM* good in the next *CM*, so that the implicit price of *DM* good is equal to one in terms of the current period *CM* good.

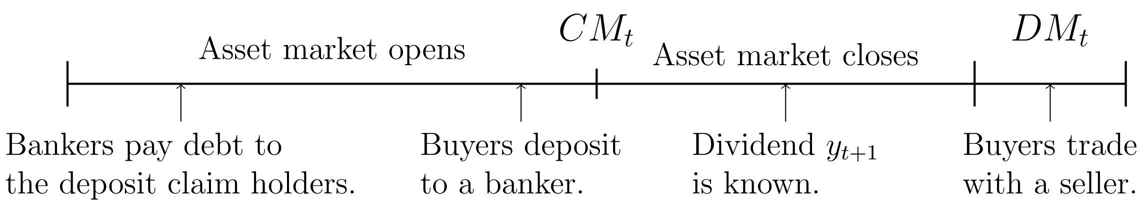
[22] Deposit claims issued by buyers or sellers cannot be used in the *DM*, since the anonymity assumption for buyers and sellers in the *DM* would be violated.

[23] I assume that counterfeiting of deposit claims is impossible.

[24] Note that all agents are subject to the same degree of limited commitment in the model.

In the economy there exists only one real asset - a divisible Lucas tree. It is endowed to buyers in the *CM* of the initial period $t = 0$, with a fixed unit supply. The Lucas tree pays off y_t units of consumption goods in period t as a dividend, and trades at the price ψ_t in terms of goods in the period t *CM*. The dividend of the Lucas tree, y_t, is an *i.i.d* random variable which can take on two possible values, $0 \leq y^\ell \leq y^h < \infty$. Let π denote the probability of a high dividend y^h, and let $\bar{y} \equiv \pi y^h + (1-\pi)y^\ell$ be the expected payoff of this random dividend.[25]

In order to introduce capital requirements I assume that there exists a government, a separate agent, that can force the bankers to maintain a minimum capital-asset ratio by states. This state-contingent capital requirement is determined ex ante and committed for the future periods. The government is benevolent, so that it chooses a capital requirement to maximize the economic welfare of the whole society.



[Figure 2. Time line]

Timing is described in Figure 2. In the beginning of the period t *CM*, all agents meet together. The previous deposit claims are paid off and the holders of the Lucas tree receive the realized dividends. The government can impose bank capital requirements on the bankers. Then a Walrasian market opens: Goods are produced, assets are traded and buyers deposit the goods to a banker and make a contingent deposit contract.[26] Perfect competition is assumed among the continuum of bankers, so that a representative banker suggests a deposit contract which maximizes the expected value of depositors.[27] Then the banker invests in the real asset, using the deposits and their own capital, which they can acquire by supplying their labor. At the end of the period t *CM*, the competitive asset market closes and the next period $t+1$ dividend of the Lucas tree is revealed to everyone.[28]

A key feature of the model is that the government can adjust the proportion of the underlying assets that can support trade in the *DM* by imposing capital requirements, because

[25] I assume $\bar{y} > 0$ to avoid multiple equilibria.

[26] Buyers can also deposit in a form of assets as the Walrasian market is open.

[27] Note that, even if buyers could directly use the real asset for the trade in the *DM*, there would be no additional benefit from such direct asset-trades because the representative banker provides the maximized contract for buyers, with zero profit for themselves.

[28] If no one knows the realization of the dividends in the *DM* then the terms of trade would depend on the expected payoff, $\bar{y}$, regardless of the state.

the proportion of bank assets assigned to bank capital holders, i.e. bankers, is not available to support trade in the *DM*.

3 Competitive Equilibrium

In the model deposit contracts between buyers and bankers are necessary, because only deposit claims issued by bankers can be accepted in the *DM*. Under perfect competition a representative banker provides a deposit contract that maximizes the expected value of buyers in equilibrium. In this respect the buyer's problem is trivial, since it is solved by the representative banker: Buyers can choose a deposit contract, but in equilibrium buyers will accept the optimal deposit contract suggested by the representative banker. The seller's problem is also trivial in this respect: Sellers can accept or reject the buyer's offer in the *DM*, but in equilibrium sellers always accept the offer since the suggested deposit contract guarantees non-negative profits for sellers. Thus, in order to construct a competitive equilibrium we focus on the representative banker's problem and an asset market clearing condition.

In equilibrium, given the solution of the representative banker's problem, the benevolent government can impose capital requirements on the bankers to maximize the social welfare of the equilibrium allocation.[29] In this section I first construct a competitive equilibrium without capital requirements. Then I explore whether strictly positive bank capital requirements can improve the welfare or not.

In equilibrium a representative banker solves the following problem in the *CM* of period t:

$$\underset{d_t, a_t, x_t^h, x_t^\ell}{Max} - d_t + \pi u(x_t^h) + (1-\pi)u(x_t^\ell) \tag{1}$$

subject to

$$d_t - \psi_t a_t + \pi\{\beta(\psi_{t+1} + y^h)a_t - x_t^h\} + (1-\pi)\{\beta(\psi + y^\ell)a_t - x_t^\ell\} \geq 0 \tag{2}$$

$$\beta(\psi_{t+1} + y^h)a_t - x_t^h \geq 0 \tag{3}$$

$$\beta(\psi_{t+1} + y^\ell)a_t - x_t^\ell \geq 0 \tag{4}$$

$$d_t, a_t, x_t^h, x_t^\ell \geq 0 \tag{5}$$

The problem (1) subject to (2)-(5) states that a state-contingent banking contract (d_t, x_t^h, x_t^ℓ) is chosen in equilibrium to maximize the expected utility of the buyers subject to the partic-

[29]Note that the benevolent government play a role as a planner in the model, since the government maximizes the welfare given the equilibrium conditions of the agents under the same degree of limited commitment.

ipation constraint (2) for the banker, and the incentive constraints (3)-(4) for the banker in each state, as well as non-negativity constraints (5).[30][31] In (1)-(5) d_t denotes the quantity of period t *CM* goods deposited by the buyer, a_t denotes the demand of the banker for asset holdings, and x_t^i represents the period t *DM* consumption level of the buyer in each state i for $i = h, \ell$. The expression on the left-hand side of (2) is the net payoff for the banker. In the *CM* of period t the banker receives an amount d_t of consumption goods, issues a deposit claim, and invests in a_t units of the real asset with a market value of $\psi_t a_t$.[32] In the next *CM* the banker earns the return of the investment, $\beta(\psi_{t+1} + y^i)a_t$ in terms of the previous *CM* goods in state i, and pays x_t^i to the holder of the deposit claim in state i. Note that we have a participation constraint for the banker, instead of a budget constraint, since the banker's labor supply is unlimited in the model. The limited commitment constraints (3)-(4) represents that the banker cannot be forced to pay more than his/her real asset holdings when deposit claims are paid off. Note that a real transfer between the states is not available because of the limited commitment assumption: With full commitment, the risk-neutral banker can always provide the expected return on the asset regardless of the realization of the state by using his/her own labor supply. In this respect the aggregate risk in the asset return cannot be diversified with limited commitment, although a risk-neutral banker can provide a state-contingent contract.[33]

The benevolent government can impose a state-contingent bank capital requirement (δ^h, δ^ℓ), given which each banker must hold bank capital that will provide at least a proportion $\delta^i \in [0, 1)$ of the asset portfolio in the state i. Since this δ^i proportion of the asset portfolio is unavailable to support its in the state i, we have additional bank capital constraints by state i:

$$\beta(\psi_{t+1} + y^h)(1 - \delta^h)a_t - x_t^h \geq 0 \tag{6}$$

$$\beta(\psi_{t+1} + y^\ell)(1 - \delta^\ell)a_t - x_t^\ell \geq 0 \tag{7}$$

where the deposit claim is only supported by a proportion $1 - \delta^i$ of the assets in the state i. The capital requirement, (δ^h, δ^ℓ), represents the proportions of assets seized for equity holders by states ex post, but it is a single capital requirement and committed ex ante: Given (δ^h, δ^ℓ), the bankers require to issue an amount of bank capital ex ante which will provide δ^i proportion of assets in state i. Thus, we can map the ex post required proportions

[30] x_t^h and x_t^ℓ are consumptions in the period t *DM*, but they are also determined in the period t *CM* because the consumptions are restricted by the value of the bank's asset portfolio.

[31] The banker can also choose a non-contingent contract as an optimal choice since a non-contingent contract is simply a special case of a state-contingent contract.

[32] Buyers can deposit an amount $\frac{d_t}{\psi_t}$ of real assets instead, since they can participate in the asset market.

[33] Krishnamurthy (2003) shows that given the scarcity of aggregate collateral, the amplification effect still exists even with state-contingent securities, because risk-sharing is restricted under limited commitment.

(δ^h, δ^ℓ) into the ex ante bank capital requirement.[34] Note that given $\delta^i \in [0, 1)$, the limited commitment constraints (3)-(4) can be replaced with the bank capital constraints (6)-(7) in the model.[35] In addition, since a bank capital requirement, (δ^h, δ^ℓ), is a choice variable of the government, so that no bank capital requirements, $\delta^h = \delta^\ell = 0$, are also feasible to be chosen in the model.

In order to characterize equilibrium the first step is to solve the problem (1) subject to (2), (5)-(7). The constraint (2) must bind, since the objective function is strictly increasing in both x_t^h and x_t^ℓ while (2) is strictly decreasing in both x_t^h and x_t^ℓ. Let $\pi\lambda^h$ and $(1-\pi)\lambda^l$ denote the multipliers associated with the incentive constraints (6)-(7), respectively. By plugging (2) into (1), we have the first-order conditions for a_t, x_t^h, and x_t^ℓ,

$$\psi_t = \pi\beta(\psi_{t+1} + y^h)\{1 + \lambda^h(1 - \delta^h)\} + (1 - \pi)\beta(\psi_{t+1} + y^\ell)\{1 + \lambda^\ell(1 - \delta^\ell)\}, \tag{8}$$

$$u^{'}(x_t^h) - 1 = \lambda^h, \tag{9}$$

$$u^{'}(x_t^\ell) - 1 = \lambda^\ell, \tag{10}$$

which can be reduced into

$$\begin{aligned} \psi_t \;=\; & \pi\beta(\psi_{t+1} + y^h)\{1 + (1 - \delta^h)(u^{'}(x_t^h) - 1)\}+ \\ & (1 - \pi)\beta(\psi_{t+1} + y^\ell)\{1 + (1 - \delta^\ell)(u^{'}(x_t^\ell) - 1)\}. \end{aligned} \tag{11}$$

The first-order condition (11) states that the asset price, the marginal cost for acquiring one unit of the asset, is the same as the expected marginal trading gain of the asset in equilibrium. The expected benefit of the asset includes the resale values, $\beta(\psi_{t+1} + y^i)$, and additional utilities from using $1 - \delta^i$ proportion of the assets as collateral.

In equilibrium a representative bank holds the entire supply of the asset, so that the asset market clears in the *CM* with

$$a_t = 1 \tag{12}$$

for $t = 0, 1, 2, \ldots$. The market clearing condition (12) states that the supply of the asset is equal to the banker's demand.

Finally, since all the utility functions are linear in the *CM*, the surplus is generated from the trade in the *DM*. So the ex ante welfare function is

[34] Given (δ^h, δ^ℓ), the ex ante capital requirement can be calculated as $\frac{\psi - d}{\psi}$ in equilibrium.

[35] Note that if $\delta^i = 0$, the bank capital constraints (6)-(7) are collapsed into the limited commitment constraints (3)-(4), respectively. For $\delta^i \in (0, 1)$, if the bank capital constraints (6)-(7) do not bind, then the limited commitment constraints (3)-(4) also do not bind, while if the bank capital constraints (6)-(7) bind then the limited commitment constraints (3)-(4) are redundant, respectively.

$$W(x_t^h, x_t^\ell) = \pi\{u(x_t^h) - x_t^h\} + (1-\pi)\{u(x_t^\ell) - x_t^\ell\} + \bar{y} \tag{13}$$

which consists of the trading gains in the *DM*, plus the expected dividend from the Lucas tree. Note that the first-best equilibrium allocation is

$$x_t^\ell = x_t^h = x^*,$$

where x^* satisfies $u^{'}(x^*) = 1$.

Definition 1. *Given* (π, y^h, y^ℓ) *and a bank capital requirement* (δ^h, δ^ℓ)*, a stationary competitive equilibrium consists of quantities* (x^h, x^ℓ)*, multipliers* $(\lambda^h, \lambda^\ell)$*, and a bounded path of asset price* ψ*, which satisfy (6)-(10), (12).*[36]

From now on I restrict the attention to stationary equilibrium allocations. Note that there are six variables $(a, x^h, x^\ell, \psi, \lambda^h, \lambda^\ell)$ to be determined in a stationary equilibrium in Definition 1, and we have six equations (6)-(10) and (12).

3.1 No Bank Capital Requirements

In this subsection I characterize the equilibrium allocations with no bank capital requirements, i.e. $\delta^h = \delta^\ell = 0$, as a benchmark. Different equilibrium cases are determined according to which of the incentive constraints (6)-(7) bind or not. I consider each of three relevant equilibrium cases: neither constraint binds; the constraint for state ℓ only binds; both constraints bind. Note that there is no equilibrium in which only the constraint for state h binds, since $y^h \geq y^\ell$ is assumed.

3.1.1 Neither constraint binds

Since $\lambda^h = \lambda^\ell = 0$ holds, we have $x^\ell = x^h = x^*$ from (9)-(10) and also have $\psi = \frac{\beta\bar{y}}{1-\beta}$ from (8). Note that the asset price is the same as its fundamental value, $\psi = \psi_f := \frac{\beta\bar{y}}{1-\beta}$, where $\frac{\beta\bar{y}}{1-\beta}$ is the expected sum of its future dividends. The quantity of bank deposits, d, is fixed as x^* in the participation constraint (2), since (2) holds with equality under perfect competition. Thus, when neither incentive constraint binds, the first-best allocation is attained. It implies that, if the supply of the asset is sufficiently large to overcome limited commitment, then the efficient allocation is achieved. Given $\delta^h = \delta^\ell = 0$, if the incentive constraint for state ℓ (7) does not bind, the incentive constraint for state h (6) also does not bind. Thus, by

[36]Since $\delta^i \in [0, 1)$, from (8)-(10) the asset price is bounded as $\psi \geq \psi_f := \frac{\beta\bar{y}}{1-\beta}$ in stationary equilibrium.

plugging $\psi = \psi^f$ into (7) we can have a necessary condition,

$$\beta y^\ell + \frac{\beta^2 \bar{y}}{1-\beta} \geq x^*, \tag{14}$$

to support the efficient allocation as an equilibrium. This condition requires that the state ℓ dividend in the next period plus the expected discounted dividends in the following periods is greater than the optimal consumption level, x^*. In equilibrium the bank capital, the asset portfolio minus bank deposits, $\psi - d$, is determined as $\psi^f - x^*$ and could be strictly positive. However, it is not costly for the banker to hold bank capital in this case since the rate of return on the asset is the same as the rate of time preference, $\frac{\psi^f+\bar{y}}{\psi^f} = \frac{1}{\beta}$.

3.1.2 Only the constraint for state ℓ binds

In this case given $\lambda^\ell > \lambda^h = 0$ we have $u'(x^h) = 1$ in (9). The incentive constraint for the state ℓ (7) and the first-order condition (8) can be rewritten as

$$\beta(\psi + y^\ell) - x^\ell = 0 \tag{15}$$

and

$$\psi = \frac{\pi\beta y^h + (1-\pi)\beta y^\ell u'(x^\ell)}{1 - \pi\beta - (1-\pi)\beta u'(x^\ell)}, \tag{16}$$

respectively. Then (ψ, x^ℓ) can be solved from the incentive constraint (15) and the first-order condition (16) in equilibrium. Since $\lambda^\ell > \lambda^h = 0$, we have $x^\ell < x^h = x^*$ from (9)-(10).[37] Note that the asset price is greater than its fundamental value, $\psi > \psi_f$, although the supply of assets is plentiful in state h. This is because the asset price still reflects the liquidity premium in state ℓ, as $u'(x^\ell) > 1$ in (16). Since a liquidity premium exists in the asset price, the rate of return on the asset is lower than the rate of time preference, $\frac{\psi+\bar{y}}{\psi} < \frac{1}{\beta}$. It implies that holding an excess of the asset is costly and so is holding bank capital. However, the bank capital, determined as $\pi\{\beta(\psi + y^h) - x^*\}$, is strictly positive in equilibrium when the inequality of the incentive constraint in state h (6) holds. In this case the value of the underlying asset is greater than the optimal consumption level, x^*, in state h, while smaller than x^* in state ℓ. So the banker issues a bank capital which provides the extra asset, $\beta(\psi + y^h) - x^*$, in state h and nothing in state ℓ, and holds it by themselves because the buyers do not need the extra asset in state h. Therefore, in equilibrium the banker takes the bank capital by providing a labor supply in the *CM* and receives the extra asset in state h

[37] This case can be generalized with a continuous distribution for dividends. If the variance of the dividend distribution is sufficiently large, then we will have a measure of h states in which the incentive constraint for the state h does not bind.

of the next *CM*.[38]

3.1.3 Both constraints bind

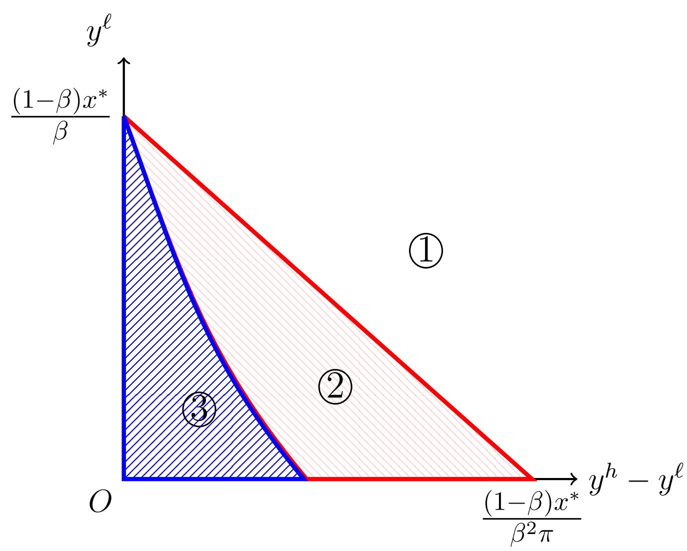
Given $\lambda^\ell > 0$, $\lambda^h > 0$, the incentive constraint for state h (6) and the first-order condition (8) can be rearranged into

$$\beta(\psi + y^h) - x^h = 0 \tag{17}$$

and

$$\psi = \frac{1 - \pi\beta u'(x^h) - (1-\pi)\beta u'(x^\ell)}{\pi\beta y^h u'(x^h) + (1-\pi)\beta y^\ell u'(x^\ell)}, \tag{18}$$

respectively. Then the incentive constraints (15), (17), and the first-order condition (18) solve for ψ, x^h, and x^ℓ in equilibrium. In (15) and (17), the consumption level in the state ℓ is lower than the consumption level in the state h, $x^\ell < x^h$, as long as $y^\ell < y^h$ holds. In (18) the asset price is greater than its fundamental value, $\psi > \psi_f$, so that holding bank capital is also costly in this case. However, bank capital is zero in equilibrium because the supply of assets is not sufficiently large to support transactions in state h as well as state ℓ.[39]



[Figure 3. Regions with no bank capital requirements($\delta^h = \delta^\ell = 0$)]

Corresponding to the three equilibrium cases, the regions 1,2, and 3 are shown in Figure 3. Note that the x-axis, $y^h - y^\ell$, represents the degree of the aggregate risk while the y-axis, y^ℓ, implies the total supply of the asset given the level of the aggregate risk, $y^h - y^\ell$. Regions

[38]Note that this is not because the banker is risk-neutral. Even if the bankers were risk-averse, this result is robust as long as the asset is plentiful in one state and scarce in the other state.

[39]Note that $\psi - d = 0$ in (2), when both incentive constraints bind.

1 and 2 are separated by the straight line which follows (14) with equality. This means that, given an aggregate risk, $y^h - y^\ell$, if the supply of assets is plentiful for the state ℓ, then the first-best allocation is feasible. The curved line between regions 2 and 3 is drawn by the points where $x^h = x^*$ just holds for the incentive constraint for the state h (17). Note that region 2 does not exist on the y-axis where $y^h = y^\ell$, since the two incentive constraints collapse into one constraint.

Except for the points on the y axis, in regions 2 and 3 equilibrium the consumption fluctuates. The aggregate risk in the asset return cannot be shared by the risk-neutral bankers when the limited commitment constraint binds. Thus, the key friction is limited commitment: A real transfer across states is restricted as long as the bankers are subject to limited commitment.[40]

3.2 Bank Capital Requirements

In this subsection I explore the circumstances under which a strictly positive capital requirement can improve the welfare of the competitive equilibrium. In the model the benevolent government can choose (δ^h, δ^ℓ) to maximize the social welfare given the equilibrium conditions of the banker's problem. Therefore, the benevolent government plays a role as a planner: In this paper the planner can be defined as one who decides the supply of deposits and equity instead of bankers to maximize the welfare, given the buyer's decision in the competitive asset market. Since both deposits and equity must be backed by the asset portfolio, the planner can simply choose the proportion of the asset portfolio for deposits and equity by states, (δ^h, δ^ℓ).

In stationary equilibrium, the incentive constraints (6)-(7), and the first-order condition (11) become

$$\beta(\psi + y^h)(1 - \delta^h) \geq x^h, \tag{19}$$

$$\beta(\psi + y^\ell)(1 - \delta^\ell) \geq x^\ell, \tag{20}$$

$$\psi = \frac{\pi\beta y^h\{1+(1-\delta^h)(u'(x^h)-1)\}+(1-\pi)\beta y^\ell\{1+(1-\delta^\ell)(u'(x^\ell)-1)\}}{1-\pi\beta\{1+(1-\delta^h)(u'(x^h)-1)\}-(1-\pi)\beta\{1+(1-\delta^\ell)(u'(x^\ell)-1)\}}, \tag{21}$$

respectively. By plugging (19)-(20) into (21), we have

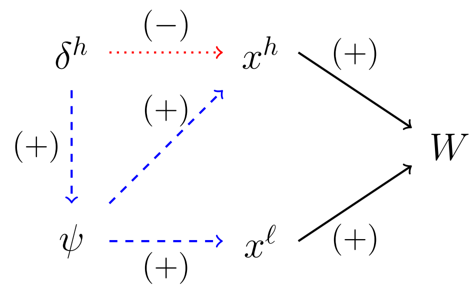
$$\psi = \frac{\pi\beta y^h\{1+(1-\delta^h)(u'(\beta(\psi+y^h)(1-\delta^h))-1)\}+(1-\pi)\beta y^\ell\{1+(1-\delta^\ell)(u'(\beta(\psi+y^\ell)(1-\delta^\ell))-1)\}}{1-\pi\beta\{1+(1-\delta^h)(u'(\beta(\psi+y^h)(1-\delta^h))-1)\}-(1-\pi)\beta\{1+(1-\delta^\ell)(u'(\beta(\psi+y^l)(1-\delta^\ell))-1)\}}, \tag{22}$$

in which ψ can be determined by capital requirements (δ^h, δ^ℓ). Then x^h is determined by (ψ, δ^h) in (19) and x^ℓ is determined by (ψ, δ^ℓ) in (20) in equilibrium.

So the benevolent government maximizes the welfare function (13) subject to the incentive

[40] If the risk-neutral banker can commit for a contingent claim, then the aggregate risk in the asset return can be diversified across the states.

constraints (19)-(20) and the first-order condition (21). Since the equilibrium allocation is efficient in region 1, I restrict our attention to regions 2 and 3 where at least one incentive constraint binds.



[Figure 4. Direct and indirect effects of capital requirements]

We can divide the impact of the capital requirement, (δ^h, δ^ℓ), on the welfare function (13) into two factors. One is the direct effect whereby a rise in δ^i, for given ψ, reduce x^i in (19) or (20), while the other is the indirect effect whereby a rise in δ^i increases x^h and x^ℓ by raising ψ in (22). For example, in case of $\delta^h > \delta^\ell = 0$, we can describe the two effects on the welfare as described in Figure 4. The dotted arrow from δ^h to x^h represents the direct effect: x^h decreases by restricting the proportion of bank liabilities in (19). The dashed arrows from δ^h to ψ and from ψ to both x^h and x^ℓ represent the indirect effect: ψ increases because the proportion of bank liabilities is reduced in the state h. Then x^h and x^ℓ goes up as ψ increases in (19) and (20). Thus the effect of capital requirements, $\delta^h > \delta^\ell = 0$, on the welfare can be described as

$$\frac{dW(x^h, x^\ell)}{d\delta^h} = \underbrace{\frac{\partial W(x^h, x^\ell)}{\partial x^h}\frac{\partial x^h(\psi, \delta^h)}{\partial \delta^h}}_{direct\, effect} + \underbrace{\{\frac{\partial W(x^h, x^\ell)}{\partial x^h}\frac{\partial x^h(\psi, \delta^h)}{\partial \psi} + \frac{\partial W(x^h, x^\ell)}{\partial x^\ell}\frac{\partial x^\ell(\psi)}{\partial \psi}\}\frac{d\psi}{d\delta^h}}_{indirect\, effect} \tag{23}$$

Note that x^ℓ is determined only by the asset price, ψ, because there is no direct effect of δ^h on x^ℓ.

Lemma 1. *Given that the both incentive constraints bind at $(\delta^h, \delta^\ell) \in [0, 1) \times [0, 1)$, if $x^\ell \geq x^h$ in equilibrium then $\frac{dW(x^h,x^\ell)}{d\delta^h} < 0$. Similarly, if $x^h \geq x^\ell$ in equilibrium then $\frac{dW(x^h,x^\ell)}{d\delta^\ell} < 0$.*

Proof. *See the appendix.*

The result of Lemma 1 implies that given any capital requirements if the consumption level of one state is less than the other, raising the capital requirement in that state cannot improve the welfare. Raising the capital requirement in one state reduces the pledgeability in that state, so the direct effect is always negative. But the indirect effect can be positive because restricting the pledgeability of the assets raises the asset prices in general equilibrium. Lemma 1 proves that restricting the equal or lower consumption state does not increase the asset price sufficiently to improve the welfare. Specifically, if $x^\ell \geq x^h$, marginal utility in state ℓ is equal or lower than the marginal utility in state h, $u^{'}(x^h) \geq u^{'}(x^\ell)$. Thus, the asset price needs to increase further to cover the loss in the marginal utility of consumption in state h by raising δ^h, but it is not feasible when $u^{'}(x^h) \geq u^{'}(x^\ell)$. This lemma is useful to narrow down the pair of optimal capital requirements in the following proofs.

3.2.1 No Aggregate Risk

We begin with no aggregate risk case, $y^h = y^\ell = \bar{y}$, in order to show that the aggregate risk is necessary for welfare-improving capital requirements. Although the incentive constraints in states h and ℓ collapse into one, we can still consider two forms of capital requirements: One is the non-contingent capital requirement, $\delta^h = \delta^\ell > 0$ while the other is a contingent capital requirement which applies with a random variable such as sun-spot: For example, a strictly positive capital requirement can be applied randomly with a probability $\pi \in (0, 1)$, although verifying the states is unavailable with the same dividends, $y^h = y^\ell = \bar{y}$.

Proposition 1. *If there is no aggregate risk, i.e.* $y^h = y^\ell$*, and the incentive constraint binds for the scarcity of assets, then the optimal capital requirement is* $(\delta^{h*}, \delta^{\ell*}) = (0, 0)$*.*

Proof. *See the appendix.*

Proposition 1 shows that if there is only one state, either constant or random capital requirements cannot be beneficial. Specifically, in case of the random contingent capital requirement, the marginal utility of consumption in the state with the requirement is the same as the marginal utility of consumption in the state without the requirement, e.g. $u^{'}(x^h) = u^{'}(x^\ell)$, because there is no aggregate risk, $y^h = y^\ell$. So restricting one unit of consumption in one state cannot raise the asset price sufficiently to increase one unit of consumption in the other state.

3.2.2 Aggregate Risk

Now we address the optimal capital requirements given the aggregate risk, $y^h > y^\ell$. Since the optimal capital requirement consists of two choice variables, δ^h and δ^ℓ, it could be complicated to find out the optimal requirement pair. In this respect Lemma 2 is helpful for us to concentrate on one variable, δ^h.

Lemma 2. *If there is aggregate risk, i.e.* $y^h > y^\ell$*, and at least one incentive constraint binds, then the optimal capital requirement in state* ℓ *is zero,* $\delta^{\ell*} = 0$*.*

Proof. *See the appendix.*

Lemma 2 shows that the optimal capital requirement in state ℓ is zero either the incentive constraint for state h binds or not. If the incentive constraint for state h does not bind, then $x^h = x^*$. Since the direct effect dominates the indirect effect for x^ℓ in the restricted state, $\delta^{\ell*} = 0$. When both constraints bind, the logic is as follows: If $x^\ell \leq x^h$ in equilibrium then we can improve the welfare by lowering δ^ℓ by Lemma 1. Given $y^h > y^\ell$ if $x^\ell > x^h$ in equilibrium, then $\delta^h > \delta^\ell$ must be imposed, so that we can improve welfare by lowering δ^h. As long as $\delta^\ell > 0$, we must return to $x^\ell \leq x^h$ case by lowering δ^h, because $x^\ell > x^h$ is impossible with $\delta^\ell > \delta^h \geq 0$ given $y^h > y^\ell$.

Provided $\delta^{\ell*} = 0$, we can define $\bar{\delta} > 0$ as the lowest δ^h where $x^\ell = x^h < x^*$ holds in equilibrium at $\delta^h = \bar{\delta}$. Note that given $y^h > y^\ell$ and $\delta^{\ell*} = 0$, $\bar{\delta}$ exists when both incentive constraints (19)-(20) bind.[41]

Let's consider the case of region 2: Suppose that the incentive constraint for state ℓ only binds. In region 2 we can define $\hat{\delta} > 0$ as the highest δ^h where the incentive constraint (19) does not bind at $\delta^h = \hat{\delta}$. Then $\hat{\delta}$ is obtained from $\beta(\hat{\psi} + y^h)(1 - \hat{\delta}) = x^*$, where $(\hat{\psi}, \hat{x^\ell})$ satisfies (20)-(21) with $x^h = x^*$ at $\delta^h = \hat{\delta}$ and $\delta^\ell = 0$. Note that given $\delta^{\ell*} = 0$, $\hat{\delta} < \bar{\delta}$ because $x^\ell < x^h = x^*$ holds at $\delta^h \in [0, \hat{\delta}]$ and $x^\ell = x^h < x^*$ is feasible only when both incentive constraints (19)-(20) bind.

Proposition 2. *If the incentive constraint for the state ℓ only binds, then the optimal capital requirement in the state h is strictly positive and $\delta^{h*} \in (\hat{\delta}, \bar{\delta})$.*

Proof. *See the appendix.*

Proposition 2 states that the optimal capital requirement in state h is strictly positive in region 2. This is because (ψ, x^ℓ) is fixed in (20)-(21) with $x^h = x^*$, so that the welfare does not change at $\delta^h \in [0, \hat{\delta}]$. However, at $\delta^h = \hat{\delta}$, the marginal cost of direct effect is zero with $\frac{\partial W(x^h, x^\ell)}{\partial x^h} = \pi\{u'(x^*) - 1\} = 0$ whereas the marginal benefit of indirect effect is strictly positive with $\frac{d\psi}{d\delta^h} > 0$ since (19) starts to bind.

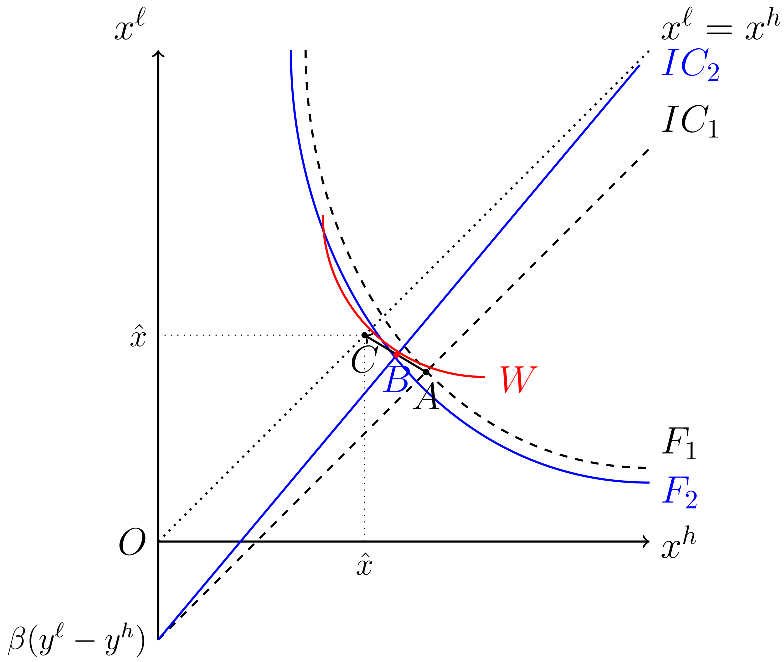
Proposition 3. *If both incentive constraints bind and the aggregate risk is sufficiently large, then the optimal capital requirement in the state h is strictly positive, $\delta^{h*} \in (0, \bar{\delta})$. If the aggregate risk is not sufficiently large, then $\delta^{h*} = 0$.*

Proof. *See the appendix.*

Proposition 3 states that the optimal capital requirement, δ^{h*}, is strictly positive when the aggregate risk and the supply of assets are sufficiently large and agents are sufficiently

[41] We have $x^\ell < x^h$ at $\delta^h = 0$. When δ^h increases to 1, x^h strictly decreases and approaches to zero while x^ℓ is still strictly positive. By the intermediate value theorem there exists $\bar{\delta} \in (0, 1)$ at which $x^\ell = x^h$ holds.

risk-averse. Given $\delta^h = \delta^l = 0$ if the aggregate risk, $y^h - y^l$, is large then the difference between marginal utility of consumptions in state ℓ and state h, $u^{'}(\tilde{x}^{\ell}) - u^{'}(\tilde{x}^{h})$, is also large, so that the indirect effect can dominate the direct effect. Moreover, as more as the supply of assets is scarce, requiring bank capital is more costly: The increase in marginal cost of holding idle assets deteriorates the welfare. Furthermore, if the buyers are more risk-averse, when the gap between x^h and x^l becomes narrow the welfare improves further by smoothing marginal utility of consumptions.



[Figure 5. Movement with a capital requirement, $\delta^h > 0$]

We can explain these results alternatively by describing the equilibrium allocation (x^h, x^h) and its movement in Figure 5 along with the welfare curve. By plugging (19)-(20) into (21) we can have the left-hand side of (24), and the right-hand side of (24) can be obtained by rearranging (21) for ψ.

$$\pi x^h\{u^{'}(x^h) + \tfrac{\delta^h}{1-\delta^h}\} + (1-\pi)x^{\ell}\{u^{'}(x^{\ell}) + \tfrac{\delta^{\ell}}{1-\delta^{\ell}}\} = \frac{\pi\beta y^h\{1+(1-\delta^h)(u^{'}(x^h)-1)\}+(1-\pi)\beta y^{\ell}\{1+(1-\delta^{\ell})(u^{'}(x^{\ell})-1)\}}{1-\pi\beta\{1+(1-\delta^h)(u^{'}(x^h)-1)\}-(1-\pi)\beta\{1+(1-\delta^{\ell})(u^{'}(x^{\ell})-1)\}} \tag{24}$$

The equilibrium condition (24) represents the feasible consumption levels in state h and ℓ, given the fixed supply of the asset. The left-hand side of (24) is strictly increasing in both x^h and x^{ℓ}, while the right-hand side of (24) is strictly decreasing in both x^h and x^{ℓ}.[42] Thus

[42] $u^{'}(x) + xu^{''}(x) = (1-\gamma)u^{'}(x) > 0$ because of $-\frac{xu^{''}(x)}{u^{'}(x)} = \gamma < 1$.

the feasibility condition (24) is downward sloping as F_1, F_2 curves in Figure 5.[43] Meanwhile, binding incentive constraints (19)-(20) can be reduced into

$$\beta(y^h - y^\ell) = \frac{x^h}{1-\delta^h} - \frac{x^\ell}{1-\delta^\ell}, \tag{25}$$

which is upward sloping as IC_1, IC_2 lines in Figure 5.

Given x^h and x^ℓ, the left-hand side of (24) is strictly decreasing in δ^i while the right-hand side of (24) is strictly increasing in δ^i. Thus, when δ^i increases the feasibility curve shifts towards the origin from F_1 to F_2: Since δ^i proportion of the assets cannot be used for transactions in state i, the consumptions x^h and x^ℓ are restricted further. On the other hand, when δ^i increases the IC line (25) rotates: If δ^h increases the IC line rotates counter-clockwise from IC_1 to IC_2 and if δ^ℓ increases, the IC line rotates clockwise. In sum, if δ^i increases, there is a cost for holding idle bank capital in the aggregate feasibility, but we can transfer a purchasing power from one state to the other.

The welfare function (13) appreciates equal consumption levels across states because of the strictly concave utility function. So the movement of equilibrium allocation (x^h, x^ℓ) towards the 45 degree line and/or away from the origin is welfare-improving. In this respect we can confirm our results with the shifts of the two curves in Figure 5. Lemma 1 shows that when $x^h \geq x^\ell$, raising δ^ℓ reduces the welfare. In Figure 5, if δ^ℓ increases, IC curve rotates clockwise and F curve moves towards the origin, so that the welfare declines. Similarly, Proposition 1 can be explained because raising $\delta^h (= \delta^\ell)$ shifts F curve towards the origin while the IC curve remains. In Proposition 3, we find out that raising δ^h at $\delta^h = \delta^\ell = 0$ can improve the welfare. In Figure 5, when δ^h increases, IC line rotates counter-clockwise while F curve moves to the origin. Thus, the allocation (x^h, x^ℓ) moves from the point A to B, and B to C and the welfare can be improved by raising δ^h when IC_1 line is located further away from 45 degree line.

The welfare improvement by imposing strictly positive capital requirements implies that there exists a pecuniary externality in the model. If all the bankers agree with issuing and buying the idle bank capital to raise the asset price in general equilibrium, then the equilibrium allocation can become constrained efficient. However, given prices the bankers are not willing to issue bank capital voluntarily because the bankers maximizes their profits under perfect competition. Note that this market failure is generated by the limited commitment of the bankers and the scarcity of collateral: Although the risk-neutral bankers can issue state-contingent claims in the model, the consumption risk is not perfectly shared because of limited commitment. Additionally, in this respect the result does not change even when two

[43] Note that given $\delta^h = \delta^\ell$, x^h is more feasible than x^ℓ because of $y^h > y^\ell$.

state-contingent assets are introduced instead of one asset with state-contingent dividends.[44]

4 Monetary Equilibrium

In real world banks invest not only in real assets, but also in nominal debts issued by the government such as money, government bonds. Unlike the real assets, the government can control the supply of money and government bonds by implementing the government policy. In this section I introduce money in the model to verify whether state-contingent monetary policy can be an alternative solution for this pecuniary externality problem.

Since we focus on the role of nominal government debt as collateral, in the model money can be held by bankers to support bank liabilities, but cannot be used directly in the DM trade. In this respect money is similar to perpetual government bonds such as Consol rather than currency in the model. The government provides money which trades at price ϕ_t in terms of goods in the CM and pays a state-contingent interest on money R^i_{t+1} in terms of money in the next CM. I assume that the government can collect a lump-sum tax from buyers in the CM, so that the government can promise the state-contingent nominal interests and support the state-contingent monetary policy by using a state-contingent tax or transfer.[45]

In period $t = 0$, money is issued with lump-sum transfer, τ_0, and in the following periods outstanding real money balances are supported by a lump-sum tax or transfer over time. So the government budget constraint for $t = 0$ is

$$\phi_0 M_0 = \tau_0,$$

and for $t = 1, 2, 3, \ldots$

$$\phi_t(M_t - M_{t-1}R^i_t) = \tau^i_t,$$

where M_t denotes the nominal quantities of money held in the private sector in period t, and τ^i_t denotes the real value of the lump-sum transfer to each buyer in state i of period t.

I assume that the real value of money supply, i.e. outstanding real money balance, is kept as V which is sufficiently small. This assumption is required to maintain the supply of the total assets scarce in the economy where the first-best allocation is infeasible and the commitment power of the government is restricted. So we have the government debt

[44] Suppose that we have two state-dependent assets in the model: One provides y^h in state h and zero in state ℓ while the other provides y^ℓ in state ℓ and zero in state h. If the total supply of state-contingent assets is insufficient to support trade, the consumption levels fluctuate. At that time capital requirements can be beneficial by adjusting the asset prices and transferring purchasing power from one state to the other.

[45] The lump-sum tax can be negative in the model and it is collected in terms of consumption goods.

constraints for $t = 0, 1, 2, \ldots$ as

$$\phi_t M_t = V. \tag{26}$$

Note that the required lump-sum transfer to maintain the constant value, V, for $t = 0$ is $\tau_0 = V$ and for state i in period $t = 1, 2, 3, \ldots$ is

$$\tau_t^i = (1 - \tfrac{\phi_{t+1}}{\phi_t} R^i)V.$$

Now we will consider a state-contingent monetary policy along with a capital requirement in the model. In the model a representative banker solves the following problem in the *CM* of period t:

$$\underset{d_t, b_t, a_t, x_t^h, x_t^\ell}{Max} - d_t + \pi u(x^h) + (1 - \pi) u(x^\ell)$$

subject to participation constraint,

$$\begin{aligned} d_t - m_t - \psi_t a_t + \pi\{\frac{\beta\phi_{t+1}}{\phi_t} R_t^h m_t + \beta(\psi_{t+1} + y^h) a_t - x_t^h\} \\ +(1 - \pi)\{\frac{\beta\phi_{t+1}}{\phi_t} R_t^\ell m_t + \beta(\psi_{t+1} + y^\ell) a_t - x_t^\ell\} \geq \quad 0 \end{aligned} \tag{27}$$

and bank capital constraints by states,

$$\{\frac{\beta\phi_{t+1}}{\phi_t} R_t^h m_t + \beta(\psi_{t+1} + y^h) a_t\}(1 - \delta^h) - x_t^h \geq 0 \tag{28}$$

$$\{\frac{\beta\phi_{t+1}}{\phi_t} R_t^\ell m_t + \beta(\psi_{t+1} + y^\ell) a_t\}(1 - \delta^\ell) - x_t^\ell \geq 0 \tag{29}$$

and non-negative constraints,

$$d_t, b_t, a_t, x_t^h, x_t^\ell \geq 0.$$

Note that m_t denotes the real quantity of money in terms of the *CM* good in period t held by the banker and R_t^i denote the nominal interest rate on money in state $i \in \{h, \ell\}$ in period t *CM*.

I focus on a stationary equilibrium where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all t and μ denote the gross inflation rate. We will restrict our attention to the cases in which the first-best allocation is infeasible. Then the participation constraint (27) and the incentive constraint for the state ℓ (29) always bind, while the incentive constraint for the state h (28) may bind or not.

The first-order conditions by m, a can be attained as

$$\frac{\mu}{\beta} = R^h \pi\{(1 - \delta^h) u'(x^h) + \delta^h\} + R^\ell (1 - \pi)\{(1 - \delta^\ell) u'(x^\ell) + \delta^\ell\}, \tag{30}$$

$$\psi = \frac{\pi\beta y^h\{1+(1-\delta^h)(u^{'}(x^h)-1)\}+(1-\pi)\beta y^\ell\{1+(1-\delta^\ell)(u^{'}(x^\ell)-1)\}}{1-\pi\beta\{1+(1-\delta^h)(u^{'}(x^h)-1)\}-(1-\pi)\beta\{1+(1-\delta^\ell)(u^{'}(x^\ell)-1)\}}, \tag{31}$$

while incentive constraints (28)-(29) can be rewritten by dropping t subscripts as

$$\{\frac{\beta}{\mu}R^h m + \beta(\psi + y^h)a\}(1-\delta^h) \geq x^h, \tag{32}$$

$$\{\frac{\beta}{\mu}R^\ell m + \beta(\psi + y^\ell)a\}(1-\delta^\ell) \geq x^\ell. \tag{33}$$

In equilibrium asset markets clear in the *CM* for all t, so that the demands of the representative banker for money and real assets are equal to the supplies of money and the Lucas tree, respectively, as

$$m_t = \phi_t M_t, \tag{34}$$

$$a_t = 1. \tag{35}$$

Definition 2. *Given the parameters (π, y^h, y^ℓ, V) and the policy variables, $(R^h, R^\ell, \delta^h, \delta^\ell)$, a stationary monetary equilibrium consists of quantities (x^h, x^ℓ), bounded paths of asset price ψ and inflation rate, μ, and multipliers (λ_1, λ_2) which solve equations (26),(30)-(35).*

Since we have six unknown variables, $(x^h, x^\ell, \psi, \mu, R^h, R^\ell)$, with four equations (30)-(33), I assume that the government sets the state-contingent nominal interest rates (R^h, R^ℓ) as policy variables. I also assume $R^\ell = 1$ for normalization.[46] Note that the nominal interest rates can be negative, i.e. $R^i \leq 1$, in the model.

4.1 Non-contingent Monetary Policy

In this subsection we describe the equilibrium cases with non-contingent monetary policy, $R^h = 1$, and no capital requirements, $\delta^h = \delta^\ell = 0$. As shown in the previous section with a Lucas tree, we also have three cases whether two incentive constraints (32)-(33) bind or not. Given $R^h = 1$, if

$$V + \beta(\psi^f + y^\ell) \geq x^* \tag{36}$$

holds, the first-best allocation is available with $x^h = x^\ell = x^*$ and $\psi = \psi^f = \frac{\beta\bar{y}}{1-\beta}$ which is equivalent to the region 1 in the previous section. By plugging (30) into (32)-(33) we can have

$$\frac{V}{\pi u'(x^h) + (1-\pi)u'(x^\ell)} + \beta(\psi + y^h) \geq x^h \tag{37}$$

and

$$\frac{V}{\pi u'(x^h) + (1-\pi)u'(x^\ell)} + \beta(\psi + y^\ell) \geq x^\ell. \tag{38}$$

[46] In the model the relative rate of return on money between states, $\frac{R^h}{R^\ell}$, affects the equilibrium allocations (x^h, x^ℓ, ψ) while the level of R^ℓ changes only the gross inflation rate, μ.

Suppose that (36) does not hold. Then (38) must bind, while (37) binds or not. If (37) also binds then we have $x^\ell < x^h < x^*$ with $\psi > \psi^f$ which is equivalent to the region 3 and (x^h, x^ℓ, ψ) is determined by (31),(37)-(38). If (37) does not bind, then we have $x^\ell < x^h = x^*$ with $\psi > \psi^f$ which is equivalent to the region 2 and (x^ℓ, ψ) is determined by (31) and (38) with $x^h = x^*$.

4.2 Contingent Monetary Policy

In this subsection we find out how state-contingent monetary policy can improve the welfare and when it could be restricted. We begin with the equilibrium case where both incentive constraints (37)-(38) bind at $\delta^h = \delta^\ell = 0$ and $R^h = 1$ in region 3, and then consider the case where only (39) binds. As shown in the previous section, by plugging the binding constraints (32)-(33) into (31) and by rearranging it with (31), we can have the feasibility condition as

$$\pi x^h u^{'}(x^h) + (1-\pi)x^\ell u^{'}(x^\ell) \quad = V + \tfrac{\beta y^h \pi u^{'}(x^h) + \beta y^\ell (1-\pi) u^{'}(x^\ell)}{1-\beta\{\pi u^{'}(x^h) + (1-\pi)u^{'}(x^\ell)\}}, \tag{39}$$

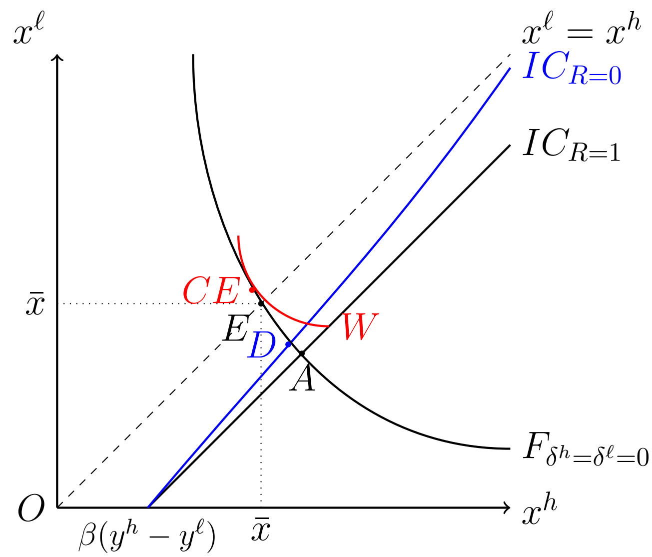
where the consumption levels (x^h, x^ℓ) are supported by the real value of money supply, V, and the Lucas tree, ψ, without any loss of holding bank capital.[47] The other equilibrium condition can be derived by subtracting (32) from (33) and using (30) as

$$x^h - x^\ell = \beta(y^h - y^\ell) + \frac{(R^h - 1)V}{R^h \pi u'(x^h) + (1-\pi)u'(x^\ell)}. \tag{40}$$

Given R^h, (x^h, x^ℓ) is uniquely determined as shown in Figure 6, because (x^h, x^ℓ) is negatively related in F curve (39) while positively related in IC curve (40).

If R^h decreases then (40) shifts leftwards, so that x^ℓ rises while x^h falls in equilibrium. That means, state-contingent monetary policy can change the equilibrium allocation along F curve. Intuitively, $R^h < 1$ implies that the rate of return on money in state h is lower than the rate of return on money in state ℓ. Therefore, the incentive constraint for state h (32) is restricted while the incentive constraint for state ℓ (33) is relaxed relatively in equilibrium. Thus, the state-contingent monetary policy can transfer a purchasing power from one state to the other as well as the state-contingent capital requirements. However, the mechanism of monetary policy is different from capital requirements: The state-contingent monetary policy redistribute the real value of money balance across states directly by changing the state-contingent taxes or transfers while the capital requirements affect the asset prices to transfer the purchasing power indirectly.

[47] Given bank capital requirements (δ^h, δ^ℓ) the terms, $\frac{\delta^h}{1-\delta^h}$ and $\frac{\delta^\ell}{1-\delta^\ell}$ in (24) reduce the feasibility of (x^h, x^ℓ).



[Figure 6. State-contingent monetary policy]

Since the equilibrium allocation moves along F curve without any cost for holding bank capital, if there is no restriction in monetary policy, the constrained efficient allocation $(x^{h*}, x^{\ell*})$ can be found when the slope of the feasibility condition (39) is equal to the slope of the welfare function (13) as

$$\frac{\partial x^\ell}{\partial x^h} \mid V = -\frac{\pi\{(1-\gamma)u'(x^{h*}) - K_1'(x^{h*}, x^{\ell*})\}}{(1-\pi)\{(1-\gamma)u'(x^{\ell*}) - K_2'(x^{h*}, x^{\ell*})\}} = -\frac{\pi\{u'(x^{h*})-1\}}{(1-\pi)\{u'(x^{\ell*})-1\}} = \frac{\partial x^{\ell*}}{\partial x^{h*}} \mid W, \qquad (41)$$

where $K(x^h, x^\ell) = \frac{\beta y^h \pi u'(x^h) + \beta y^\ell (1-\pi) u'(x^\ell)}{1-\beta\{\pi u'(x^h) + (1-\pi)u'(x^\ell)\}}$. Note that $(x^{h*}, x^{\ell*})$ is located in the upper left side of 45 degree line because the slope of F curve (39) is steeper than the slope of the welfare curve (13) at the point E where $x^h = x^\ell$ in Figure 6.[48]

Lemma 3. *For (x^h, x^ℓ) on F curve, if $x^h > x^{h*}$ and $x^\ell < x^{\ell*}$, then the slope of F curve (39) is steeper than the slope of the welfare curve (13).*

Proof. *See the appendix.*

Then we can verify whether the monetary policy is limited or not, by comparing the equilibrium allocation at $R^h = 0$ with the constrained efficient allocation. The limit of the state-contingent monetary policy is $R^h = 0$ because the total real value of money, V, supports only the consumption level in the state ℓ at $R^h = 0$. Define $(\bar{x}^h, \bar{x}^\ell, \bar{\psi}, \bar{\mu})$ as an equilibrium allocation at $R^h = 0$ in region 3. If $\bar{x}^h \leq x^{h*}$ and $\bar{x}^\ell \geq x^{\ell*}$, then the efficient allocation is feasible with $R^h \in [0, 1)$. Otherwise, the state-contingent monetary policy is

[48] Since $y^h > y^\ell$, $-K_1'(\bar{x}, \bar{x}) > -K_2'(\bar{x}, \bar{x})$ holds.

limited because V is not sufficiently large to reduce the gap between x^h and x^ℓ as shown in the point D in Figure 6.

In region 2 the total supply of assets is still scarce with $V + \beta(\psi^f + y^\ell) < x^*$, but only the incentive constraint (32) binds. If there is $R^{h*} \in [0, 1)$ that satisfies with $\frac{V}{R^{h*}\pi+(1-\pi)} + \beta(\psi^f + y^\ell) = x^*$, and $\frac{R^{h*}V}{R^{h*}\pi+(1-\pi)} + \beta(\psi^f + y^h) \geq x^*$ holds at $R^h = R^{h*}$ then the first-best allocation, $x^\ell = x^h = x^*$, can be achieved by the state-contingent monetary policy, $R^h = R^{h*}$. Otherwise, the first-best allocation is infeasible and there are three cases available at $R^h = 0$. Define $(\breve{x}^h, \breve{x}^\ell, \breve{\psi}, \breve{\mu})$ as an equilibrium allocation at $R^h = 0$ in region 2. First, if both incentive constraints (32)-(33) bind and $\breve{x}^h \leq x^{h*}$ and $\breve{x}^\ell \geq x^{\ell*}$ at $R^h = 0$, then the constrained efficient allocation is feasible with $R^h \in [0, 1)$. Second, if both incentive constraints (32)-(33) bind, but $\breve{x}^h > x^{h*}$ and $\breve{x}^\ell < x^{\ell*}$ at $R^h = 0$, then the state-contingent monetary policy is limited. Finally, if the incentive constraint (32) does not bind while (33) binds with $x^\ell < x^h = x^*$ at $R^h = 0$, then the state-contingent monetary policy is also restricted. Note that the second case is similar to the case where monetary policy is limited in region 3.

Proposition 4. *In region 2 and 3 when the state-contingent monetary policy is limited with small V, the state-contingent capital requirements can improve the welfare of the equilibrium allocation further if γ is sufficiently large and/or the gap between y^h and y^ℓ is sufficiently large.*

Proof. *See the appendix.*

The Proposition 4 shows that when state-contingent monetary policy is restricted with small V, capital requirements can be still effective to transfer a purchasing power across states. It is because capital requirements can affect the whole asset portfolio of the banker including the Lucas tree while monetary policy is only available within the fiscal limit, V.

In sum, capital requirements are beneficial when the aggregate risk is large and the total supply of the asset is not too scarce because of the cost of holding idle capital. On the other hand, monetary policy is always beneficial given the aggregate risk because there is no cost for adjusting the marginal rate of substitution by collecting the lump-sum taxes within the fiscal limit. Therefore, given the asset scarcity if the aggregate risk is small and the supply of fiat money is sufficiently large, then monetary policy can achieve the constrained efficiency and capital requirements are not necessary. However, if the aggregate risk is sufficiently large, but the supply of fiat money is small, then monetary policy is restricted and capital requirements can be beneficial as shown in Proposition 4.

5 Conclusion

This paper sheds light on the role of bank capital requirements as a macro-prudential policy that stabilize the business cycles by adjusting the pledgeability of assets. This implication is consistent with recent empirical studies in which macro-prudential policy tools are shown as effective in stabilizing the business cycles and the credit growth. Lim et al. (2011) find that several macro-prudential tools such as the Loan-to-Value ratio cap, dynamic provisioning, and the counter-cyclical buffer, can reduce the pro-cyclicality of credit growth by using the 2011 IMF survey data. Akinci and Olmstead-Rumsey (2017) develop a new index of macro-prudential policies in 57 countries and show that macro-prudential policy variables exert a negative effect on bank credit growth with a dynamic panel data model. Although their results are silent in welfare issues, but they show the possibility of stabilizing the business cycles by implementing the macro-prudential policy tools. Specifically, this paper can contribute to this growing literature by providing a relevant justification for welfare improvement with a theoretical model in which the cost of holding capital is endogenously chosen.

This paper takes steps to understand the role of bank capital requirements for efficient liquidity provision. In the model when banks issue inside money under perfect competition, they do not internalize the effect of issuing inside money on the asset prices in general equilibrium. This paper shows that bank capital requirements can correct the pecuniary externality. However, we need to study further how other policy tools or regulations should be addressed to correct this externality. Moreover, this paper focuses mainly on the liquidity creation role of banks to evaluate bank capital requirements. Therefore, capital requirements need to be evaluated along with the other functions of banks such as financial intermediation, liquidity insurance, and monitoring in near future.

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A Proofs

Lemma 1. *Given the both incentive constraints bind at $(\delta^h, \delta^\ell) \in [0,1) \times [0,1)$, if $x^\ell \geq x^h$ in equilibrium then $\frac{dW(x^h,x^\ell)}{d\delta^h} < 0$. Similarly, if $x^h \geq x^\ell$ in equilibrium then $\frac{dW(x^h,x^\ell)}{d\delta^\ell} < 0$.*

Proof. *Given $(\bar{\delta}^h, \bar{\delta}^\ell) \in [0,1) \times [0,1)$, define $(\bar{x}^h, \bar{x}^\ell, \bar{\psi})$ to be the equilibrium allocation which satisfies the equilibrium conditions (19)-(20) and $\bar{x}^\ell \geq \bar{x}^h$. The partial derivatives*[49] *at the equilibrium allocation, evaluated at $(\delta^h, \delta^\ell) = (\bar{\delta}^h, \bar{\delta}^\ell)$, are*

$$\begin{aligned}
\tfrac{\partial x^h(\psi,\delta^h)}{\partial \psi} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= \beta(1-\bar{\delta}^h) > 0, \\
\tfrac{\partial x^\ell(\psi)}{\partial \psi} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= \beta(1-\bar{\delta}^\ell) > 0, \\
\tfrac{\partial x^h(\psi,\delta^h)}{\partial \delta^h} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= -\beta(\bar{\psi}+y^h) = -\tfrac{\bar{x}^h}{1-\bar{\delta}^h} < 0, \\
\tfrac{\partial W(x^h,x^\ell)}{\partial x^h} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= \pi\{u'(\bar{x}^h) - 1\} > 0, \\
\tfrac{\partial W(x^h,x^\ell)}{\partial x^\ell} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= (1-\pi)\{u'(\bar{x}^\ell) - 1\} > 0.
\end{aligned} \tag{A.1}$$

Thus the only difficult derivative is $\frac{d\psi}{d\delta^h}$. In order to determine $\frac{d\psi}{d\delta^h}$, define the continuous and differentiable function, $f(\cdot)$ by rearranging (22) as

$$\begin{aligned}
f(\psi,\delta^h) \;\equiv\;& \psi - \pi\beta(\psi+y^h)\{1+(1-\delta^h)(u'(\beta(\psi+y^h)(1-\delta^h))-1)\} \\
&-(1-\pi)\beta(\psi+y^\ell)\{1+(1-\delta^\ell)(u'(\beta(\psi+y^\ell)(1-\delta^\ell))-1)\}.
\end{aligned}$$

Then the effect of δ^h on ψ, $\frac{d\psi}{d\delta^h}$,can be determined by taking total derivative of $f(\psi,\delta^h)=0$, yielding $\frac{d\psi}{d\delta^h} = -\frac{\frac{\partial f(\psi,\delta^h)}{\partial \delta^h}}{\frac{\partial f(\psi,\delta^h)}{\partial \psi}}$ where we have

$$\begin{aligned}
\tfrac{\partial f(\psi,\delta^h)}{\partial \delta^h} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= \pi\beta(\bar{\psi}+y^h)\{u'(\beta(\bar{\psi}+y^h)(1-\bar{\delta}^h)) + \beta(\bar{\psi}+y^\ell)(1-\bar{\delta}^h)u''(\beta(\bar{\psi}+y^\ell)(1-\bar{\delta}^h)) - 1\} \\
&= \tfrac{\pi\bar{x}^h}{1-\bar{\delta}^h}\{u'(\bar{x}^h) + \bar{x}^h u''(\bar{x}^h) - 1\},
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
\tfrac{\partial f(\psi,\delta^h)}{\partial \psi} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= 1 - \pi\beta\{(1-\bar{\delta}^h)u'(\beta(\bar{\psi}+y^h)(1-\bar{\delta}^h)) + \bar{\delta}^h + \beta(\bar{\psi}+y^h)(1-\bar{\delta}^h)u''(\beta(\bar{\psi}+y^h)(1-\bar{\delta}^h))\} \\
&\quad -(1-\pi)\beta\{(1-\bar{\delta}^\ell)u'(\beta(\bar{\psi}+y^\ell)(1-\bar{\delta}^\ell)) + \bar{\delta}^\ell + \beta(\bar{\psi}+y^\ell)(1-\bar{\delta}^\ell)u''(\beta(\bar{\psi}+y^\ell)(1-\bar{\delta}^\ell))\} \\
&= 1 - \pi\beta[(1-\bar{\delta}^h)\{u'(\bar{x}^h) + \bar{x}^h u''(\bar{x}^h)\} + \bar{\delta}^h] - (1-\pi)\beta[(1-\bar{\delta}^\ell)\{u'(\bar{x}^\ell) + \bar{x}^h u''(\bar{x}^\ell)\} + \bar{\delta}^\ell].
\end{aligned} \tag{A.3}$$

From (A.2)-(A.3) we have the total effect of capital requirement, δ^h, on the asset price, ψ,

$$\begin{aligned}
\frac{d\psi}{d\delta^h} \Big|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= \frac{\frac{\pi\bar{x}^h}{1-\bar{\delta}^h}\{1 - u'(\bar{x}^h) - \bar{x}^h u''(\bar{x}^h)\}}{1 - \pi\beta[(1-\bar{\delta}^h)\{u'(\bar{x}^h) + \bar{x}^h u''(\bar{x}^h)\} + \bar{\delta}^h] - (1-\pi)\beta[(1-\bar{\delta}^\ell)\{u'(\bar{x}^\ell) + \bar{x}^h u''(\bar{x}^\ell)\} + \bar{\delta}^\ell]} \\
&= \frac{\frac{\pi\bar{x}^h}{1-\bar{\delta}^h}\{1 - g(\bar{x}^h)\}}{(1-\beta) + \pi\beta(1-\bar{\delta}^h)\{1-g(\bar{x}^h)\} + (1-\pi)\beta(1-\bar{\delta}^\ell)\{1-g(\bar{x}^\ell)\}},
\end{aligned} \tag{A.4}$$

[49] Note that these are right-hand derivatives, for $\delta^h = 0 + \varepsilon$ as $\varepsilon \to 0$.

where $g(x) \equiv u^{'}(x) + xu^{''}(x)$ *is defined. Note that the denominator* $1 - \pi\beta[(1-\bar{\delta}^h)\{u^{'}(\bar{x}^h) + \bar{x}^h u^{''}(\bar{x}^h)\} + \bar{\delta}^h] - (1-\pi)\beta[(1-\bar{\delta}^\ell)\{u^{'}(\bar{x}^\ell) + \bar{x}^h u^{''}(\bar{x}^\ell)\} + \bar{\delta}^\ell] > 0$ *because the asset price in (21) must be strictly positive in equilibrium. By plugging (A.1) and (A.4) into (23), we have*

$$\begin{aligned}\frac{dW(x^h,x^\ell)}{d\delta^h}\,|_{(\bar{\delta}^h,\bar{\delta}^\ell)} &= \pi(u^{'}(\bar{x}^h)-1)\{-\frac{\bar{x}^h}{1-\bar{\delta}^h} + \beta(1-\bar{\delta}^h)\frac{d\psi}{d\delta^h}\} + (1-\pi)(u^{'}(\bar{x}^\ell)-1)\beta(1-\bar{\delta}^\ell)\frac{d\psi}{d\delta^h} \\ &= -\frac{\pi\bar{x}^h}{1-\bar{\delta}^h}(u^{'}(\bar{x}^h)-1) + \{\pi\beta(1-\bar{\delta}^h)(u^{'}(\bar{x}^h)-1) + (1-\pi)\beta(1-\bar{\delta}^\ell)(u^{'}(\bar{x}^\ell)-1)\}\frac{d\psi}{d\delta^h} \\ &= \frac{\pi\bar{x}^h\{1-g(\bar{x}^h)\}}{1-\bar{\delta}^h}[-\frac{u^{'}(\bar{x}^h)-1}{1-g(\bar{x}^h)} + \frac{\pi\beta(1-\bar{\delta}^h)(u^{'}(\bar{x}^h)-1)+(1-\pi)\beta(1-\bar{\delta}^\ell)(u^{'}(\bar{x}^\ell)-1)}{(1-\beta)+\pi\beta(1-\bar{\delta}^h)\{1-g(\bar{x}^h)\}+(1-\pi)\beta(1-\bar{\delta}^\ell)\{1-g(\bar{x}^\ell)\}}].\end{aligned} \tag{A.5}$$

Note that if $g(\bar{x}^h) \geq 1$ *then* $\frac{d\psi}{d\delta^h}\,|_{(\bar{\delta}^h,\bar{\delta}^\ell)} \leq 0$ *in (A.4), so that* $\frac{dW(x^h,x^\ell)}{d\delta^h}\,|_{(\bar{\delta}^h,\bar{\delta}^\ell)} < 0$*. Suppose* $g(\bar{x}^h) < 1$*. Then if* $\frac{u^{'}(\bar{x}^h)-1}{1-g(\bar{x}^h)} \geq \frac{u^{'}(\bar{x}^\ell)-1}{1-g(\bar{x}^\ell)}$ *then the value inside the large parentheses in the third row of (A.5) is negative, so that* $\frac{dW(x^h,x^\ell)}{d\delta^h}\,|_{(\bar{\delta}^h,\bar{\delta}^\ell)} < 0$*. Given* $\bar{x}^\ell \geq \bar{x}^h$*, we need to show that* $\frac{u^{'}(\bar{x}^h)-1}{1-g(\bar{x}^h)} \geq \frac{u^{'}(\bar{x}^\ell)-1}{1-g(\bar{x}^\ell)}$ *holds. Since* $g^{'}(x) < 0$*, we have* $h^{'}(x) \leq 0$ *where* $h(x) \equiv \frac{u^{'}(x)-1}{1-g(x)}$*. QED*

Proposition 1. *If there is no aggregate risk, i.e.* $y^h = y^\ell$*, and the incentive constraint binds for the scarcity of assets, then the optimal capital requirement is* $(\delta^{h*}, \delta^{\ell*}) = (0,0)$*.*

Proof. *Since* $y^h = y^\ell$*, from (19)-(20)* $\delta^h > \delta^\ell$ *iff* $x^h < x^\ell$*,* $\delta^h < \delta^\ell$ *iff* $x^h < x^l$*, and* $\delta^h = \delta^\ell$ *iff* $x^h = x^\ell$*. Given any* $(\delta^h,\delta^\ell) \in (0,1)\times(0,1)$ *in equilibrium* $x^\ell \geq x^h$ *or* $x^\ell \leq x^h$ *holds. By Lemma 1 the welfare improves by decreasing* δ^h *or* δ^ℓ*, respectively. Thus the welfare is maximized at* $(\delta^h,\delta^\ell) = (0,0)$*. Note that this proof also applies for any* $\pi \in (0,1)$*. QED*

Lemma 2. *If there is aggregate risk, i.e.* $y^h > y^\ell$*, and at least one incentive constraint binds, then the optimal capital requirement in the state l is zero,* $\delta^{\ell*} = 0$*.*

Proof. *Suppose that only the constraint for state* ℓ *binds. The effect of* δ^ℓ *on the welfare can be described as*

$$\frac{dW(x^h,x^\ell)}{d\delta^\ell} = \frac{\partial W(x^h,x^\ell)}{\partial x^\ell}\frac{\partial x^\ell(\psi,\delta^\ell)}{\partial \delta^\ell} + \{\frac{\partial W(x^h,x^\ell)}{\partial x^\ell}\frac{\partial x^\ell(\psi,\delta^\ell)}{\partial \psi} + \frac{\partial W(x^h,x^\ell)}{\partial x^h}\frac{\partial x^h(\psi)}{\partial \psi}\}\frac{d\psi}{d\delta^\ell}. \tag{A.6}$$

Since $x^\ell < x^h = x^*$*,* $\frac{\partial W(x^h,x^\ell)}{\partial x^h} = 0$*, while* $\frac{\partial W(x^h,x^\ell)}{\partial x^\ell} > 0$*. Then* $\frac{dW(x^h,x^\ell)}{d\delta^\ell} < 0$ *because*

$$\frac{\partial x^\ell(\psi,\delta^\ell)}{\partial \delta^\ell} + \frac{\partial x^\ell(\psi,\delta^\ell)}{\partial \psi}\frac{d\psi}{d\delta^\ell} = -\frac{x^\ell}{1-\delta^\ell} + \beta(1-\delta^\ell)\frac{\frac{(1-\pi)x^\ell}{1-\delta^\ell}\{1-g(x^\ell)\}}{(1-\beta)+(1-\pi)\beta(1-\delta^\ell)\{1-g(x^\ell)\}} < 0 \tag{A.7}$$

where $g(x) \equiv u^{'}(x) + xu^{''}(x)$ *is defined. Now suppose that both constraints bind. For any given* $\delta_0^h \geq 0$ *and* $\delta_0^\ell > 0$*, if* $x^\ell \leq x^h$ *in equilibrium then* $\frac{dW(x^h,x^\ell)}{d\delta^\ell} < 0$ *holds by Lemma 1. So* $\delta_0^\ell > 0$ *is suboptimal. For given* $\delta_0^h \geq 0$ *and* $\delta_0^\ell > 0$*, if* $x^\ell > x^h$ *in equilibrium then it must be* $\delta_0^h > \delta_0^\ell > 0$ *because*

$$\frac{x^h}{1-\delta_0^h} - \frac{x^\ell}{1-\delta_0^\ell} = \beta(y^h - y^\ell) \tag{A.8}$$

holds in equilibrium from (19)-(20) with $y^h > y^\ell$. Note that (A.8) describes the difference between x^h and x^ℓ, given δ^h and δ^ℓ. So when we have $x^\ell > x^h$ in equilibrium with $\delta_0^h > \delta_0^\ell > 0$, there always exists $\delta_1^h > \delta_0^\ell > 0$ which satisfies $x^h = x^\ell$ in

$$\frac{x^h}{1-\delta_1^h} - \frac{x^\ell}{1-\delta_0^\ell} = \beta(y^h - y^\ell). \tag{A.9}$$

By Lemma 1, $\frac{dW(x^h,x^\ell)}{d\delta^h} < 0$ holds for $\delta^h \in (\delta_1^h, \delta_0^h)$ so that we can improve welfare by lowering δ^h into δ_1^h. Then we have $x^\ell = x^h$ with $\delta_1^h > \delta_0^\ell > 0$, so that $\frac{dW(x^h,x^\ell)}{d\delta^\ell} < 0$ holds again and $\delta_0^\ell > 0$ is suboptimal. This process can repeat until δ^ℓ reaches zero, so $\delta^{\ell*} = 0$. QED

Proposition 2. *If the incentive constraint for the state ℓ only binds, then the optimal capital requirement in the state h is strictly positive,$\delta^{h*} \in (\hat{\delta}, \bar{\delta})$.*

Proof. *Suppose that an equilibrium allocation (x^h, x^ℓ, ψ) satisfies with (19)-(21) at $\delta^h \in [0, \hat{\delta})$, $\delta^\ell = 0$. Then we have $x^h = x^*$ without binding. In order to consider the effect of capital requirement in the state h on the welfare, $\frac{dW}{d\delta^h}$, we can describe the two effects on welfare as same as (23). Since the incentive constraint for the state h does not bind, the marginal utility effect of x^h on the welfare at $\delta^h \in [0, \hat{\delta})$ is zero as $\frac{\partial W(x^h,x^\ell)}{\partial x^h} |_{(\delta^h,0)} = \pi\{u'(x^*) - 1\} = 0$. Moreover, since the incentive constraint for the state h does not bind with $x^h = x^*$, $\frac{d\psi}{d\delta^h} = 0$ in (21). Thus $\frac{dW(x^h,x^\ell)}{d\delta^h} = 0$.*

Now suppose an equilibrium allocation $(\hat{x}^h, \hat{x}^\ell, \hat{\psi})$ satisfies with (19)-(21) at $\delta^h = \hat{\delta}$, $\delta^\ell = 0$. Then we still have $\hat{x}^h = x^*$, but it will bind when δ^h increases. We still have $\frac{\partial W(x^h,x^\ell)}{\partial x^h} |_{(\hat{\delta},0)} = \pi\{u'(x^*) - 1\} = 0$ since $\hat{x}^h = x^*$. However, $\frac{d\psi}{d\delta^h}$ can be changed because both incentive constraints bind when δ^h increases. Given $\delta^h = \hat{\delta}$, $\delta^\ell = 0$, (A.2) and (A.3) can be rewritten as

$$\begin{aligned}
\frac{\partial f(\psi,\delta^h)}{\partial \delta^h} |_{(\hat{\delta},0)} &= \pi\beta(\hat{\psi}+y^h)\{u'(\beta(\hat{\psi}+y^h)(1-\hat{\delta})) + \beta(\hat{\psi}+y^\ell)(1-\hat{\delta})u''(\beta(\hat{\psi}+y^\ell)(1-\hat{\delta})) - 1\} \\
&= \frac{\pi\hat{x}^h}{1-\hat{\delta}}\{u'(\hat{x}^h) + \hat{x}^h u''(\hat{x}^h) - 1\} = \frac{\pi x^*}{1-\hat{\delta}}\{u'(x^*) + x^* u''(x^*) - 1\} < 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f(\psi,\delta^h)}{\partial \psi} |_{(\hat{\delta},0)} &= 1 - \pi\beta\{(1-\hat{\delta})u'(\beta(\hat{\psi}+y^h)(1-\hat{\delta})) + \hat{\delta} + \beta(\hat{\psi}+y^h)(1-\hat{\delta})u''(\beta(\hat{\psi}+y^h)(1-\hat{\delta}))\} \\
&\quad -(1-\pi)\beta\{u'(\beta(\hat{\psi}+y^\ell)) + \beta(\hat{\psi}+y^\ell)u''(\beta(\hat{\psi}+y^\ell))\} \\
&= 1 - \pi\beta\{(1-\hat{\delta})u'(\hat{x}^h) + \hat{\delta} + \hat{x}^h u''(\hat{x}^h)\} - (1-\pi)\beta\{u'(\hat{x}^\ell) + \hat{x}^\ell u''(\hat{x}^\ell)\} \\
&= 1 - \beta\{\pi + (1-\pi)u'(\hat{x}^\ell)\} - \pi\beta x^* u''(x^*) - (1-\pi)\beta\hat{x}^\ell u''(\hat{x}^\ell)\} > 0.
\end{aligned} \tag{A.10}$$

Note that $1 - \beta\{\pi + (1-\pi)u'(\tilde{x}^\ell)\} > 0$ in the third row of (A.10) because the asset price must be strictly positive in equilibrium and $\hat{\psi} = \frac{\pi\beta y^h + (1-\pi)\beta y^\ell u'(\hat{x}^\ell)}{1-\beta\{\pi+(1-\pi)u'(\hat{x}^\ell)\}}$ at $\delta^h = \hat{\delta}$, $\delta^\ell = 0$ from (21). Thus we have $\frac{d\psi}{d\delta^h} |_{(\hat{\delta},0)} > 0$. Then we have $\frac{dW(x^h,x^\ell)}{d\delta^h} |_{(\hat{\delta},0)} = \frac{\partial W(x^h,x^\ell)}{\partial x^\ell}\frac{\partial x^\ell(\psi)}{\partial \psi}\frac{d\psi}{d\delta^h} = (1-$

$\pi)\{u^{'}(\hat{x}^\ell) - 1\}\beta\frac{d\psi}{d\delta^h} > 0$ *in (23). Finally, as we discuss in the proof of Proposition 2, given* $\delta^\ell = 0$ *if* $\delta^h \geq \bar{\delta} > 0$ *we have* $x^h \leq x^\ell$ *in equilibrium. By Lemma 1, we have* $\frac{dW(x^h,x^\ell)}{d\delta^h} < 0$ *so that* $\delta^h \geq \bar{\delta}$ *is suboptimal. QED*

Proposition 3. *If the aggregate risk is sufficiently large and both incentive constraints bind, then the optimal capital requirement in the state h is strictly positive,*$\delta^{h*} \in (0, \bar{\delta})$*. If the aggregate risk is not sufficiently large then* $\delta^{h*} = 0$.

Proof. *Given* $\delta^\ell = 0$*, if* $\delta^h \geq \bar{\delta} > 0$ *we have* $x^h \leq x^\ell$ *in equilibrium from (19)-(20). By Lemma 1, we have* $\frac{dW(x^h,x^\ell)}{d\delta^h} < 0$ *so that* $\delta^h \geq \bar{\delta}$ *is suboptimal. Now suppose that* $\delta^h = \tilde{\delta} \in [0, \bar{\delta})$*. Given* $\delta^h = \tilde{\delta}$ *and* $\delta^\ell = 0$ *we have an equilibrium allocation* $(\tilde{x}^h, \tilde{x}^\ell, \tilde{\psi})$*, which satisfies with (19)-(21) at* $\delta^h = \tilde{\delta}$ *and* $\delta^\ell = 0$ *and* $\tilde{x}^h < x^*$*. As we describe in the proof of Lemma 1, there are two effects on welfare by raising* δ^h *as (23). The partial derivatives at the equilibrium allocation, evaluated at* $\delta^h = \tilde{\delta}$*, are*

$$\begin{aligned}
\tfrac{\partial x^h(\psi,\delta^h)}{\partial \psi} \mid_{(\tilde{\delta},0)} &= \beta(1-\tilde{\delta}), \\
\tfrac{\partial x^\ell(\psi)}{\partial \psi} \mid_{(\tilde{\delta},0)} &= \beta, \\
\tfrac{\partial x^h(\psi,\delta^h)}{\partial \delta^h} \mid_{(\tilde{\delta},0)} &= -\beta(\tilde{\psi}+y^h) = -\tfrac{\tilde{x}^h}{1-\tilde{\delta}}, \\
\tfrac{\partial W(x^h,x^\ell)}{\partial x^h} \mid_{(\tilde{\delta},0)} &= \pi\{u^{'}(\tilde{x}^h)-1\}, \\
\tfrac{\partial W(x^h,x^\ell)}{\partial x^\ell} \mid_{(\tilde{\delta},0)} &= (1-\pi)\{u^{'}(\tilde{x}^\ell)-1\}.
\end{aligned} \tag{A.11}$$

Then the effect of δ^h *on* ψ, $\frac{d\psi}{d\delta^h} \mid_{(\tilde{\delta},0)}$*, can be determined by evaluating at* $\delta^h = \tilde{\delta}$ *and* $\delta^\ell = 0$ *as*

$$\frac{d\psi}{d\delta^h} \mid_{(\tilde{\delta},0)} = \frac{\frac{\pi\tilde{x}^h}{1-\tilde{\delta}}\{1-(1-\gamma)u^{'}(\tilde{x}^h)\}}{1-(1-\gamma)\beta\{\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\}}. \tag{A.12}$$

Thus, by plugging (A.11)-(A.12) into (23), the effect of δ^h *on the welfare at* $\delta^h = \tilde{\delta}$ *can be described as*

$$\begin{aligned}
\tfrac{\partial W(x^h,x^\ell)}{\partial \delta^h} \mid_{\delta^h=0} \quad &= \quad \pi\{u^{'}(\tilde{x}^h)-1\}(-\tfrac{\tilde{x}^h}{1-\tilde{\delta}} + \beta(1-\tilde{\delta})\tfrac{d\psi}{d\delta^h}) + (1-\pi)\{u^{'}(\tilde{x}^\ell)-1\}\beta\tfrac{d\psi}{d\delta^h} \\
&= \quad \underbrace{\frac{\pi\tilde{x}^h}{1-\tilde{\delta}}\{1-u^{'}(\tilde{x}^h)\}}_{direct\ effect} + \underbrace{\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)-1\right]\frac{d\psi}{d\delta^h}}_{indirect\ effect} \\
&= \quad \tfrac{\pi\tilde{x}^h}{1-\tilde{\delta}}\left[\{1-u^{'}(\tilde{x}^h)\} + \frac{\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)-1\right]\{1-(1-\gamma)u^{'}(\tilde{x}^h)\}}{1-(1-\gamma)\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\right]}\right].
\end{aligned} \tag{A.13}$$

Note that $1 - \beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\right] > 0$ *because the asset price must be strictly positive in equilibrium and* $\tilde{\psi} = \frac{\pi\beta y^h\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)\beta y^\ell u^{'}(\tilde{x}^\ell)}{1-\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\right]}$ *at* $\delta^h = \tilde{\delta}$ *and* $\delta^\ell = 0$ *in (21). Since* $1 - \beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\right] > 0$*, we need a necessary condition,* $1-(1-\gamma)u^{'}(\tilde{x}^h) > 0$*, in order to have a positive indirect effect. So let me assume* *that $\gamma > 1 - \frac{1}{u^{'}(\tilde{x}^h)}$ as we assume in the proof of Lemma 1. The main point of this proof is that the positive indirect effect can dominate the negative direct effect to improve welfare. In the second row of (A.13), by raising δ^h we sacrifice $\frac{\pi\tilde{x}^h}{1-\tilde{\delta}}$ expected trade with marginal utility of $u^{'}(\tilde{x}^h)-1$, but earn $\beta\frac{d\psi}{d\delta^h}$ trade with the greater marginal utility of $\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)-1$ as the asset price increases. We will show that the indirect effect is greater than direct effect as*

$$\frac{\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)-1\right]\{1-(1-\gamma)u^{'}(\tilde{x}^h)\}}{1-(1-\gamma)\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\right]} \geq u^{'}(\tilde{x}^h)-1. \quad \text{(A.14)}$$

By rearranging (A.14) we have

$$\frac{u^{'}(\tilde{x}^\ell)-1}{u^{'}(\tilde{x}^h)-1} \geq 1+\frac{(1-\beta)}{\beta\gamma(1-\pi)}+\frac{\pi\tilde{\delta}}{1-\pi} \quad \text{(A.15)}$$

Since $\frac{u^{'}(\tilde{x}^\ell)-1}{u^{'}(\tilde{x}^h)-1} > \frac{u^{'}(\tilde{x}^\ell)}{u^{'}(\tilde{x}^h)}$ holds, with $\frac{u^{'}(\tilde{x}^\ell)}{u^{'}(\tilde{x}^h)} = (\frac{\tilde{x}^h}{\tilde{x}^\ell})^\gamma$ we can rewrite the sufficient condition (A.15) as

$$1+\frac{\beta(y^h-y^\ell)}{\tilde{x}^\ell} \geq \left[1+\frac{(1-\beta)}{\beta\gamma(1-\pi)}+\frac{\pi\tilde{\delta}}{1-\pi}\right]^{\frac{1}{\gamma}}. \quad \text{(A.16)}$$

where $\tilde{x}^\ell$ is determined by $\tilde{\psi} = \frac{\pi\beta y^h\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)\beta y^\ell u^{'}(\tilde{x}^\ell)}{1-\beta\left[\pi\{(1-\tilde{\delta})u^{'}(\tilde{x}^h)+\tilde{\delta}\}+(1-\pi)u^{'}(\tilde{x}^\ell)\right]}$ and $\tilde{x}^\ell = \beta(\tilde{\psi}+y^\ell)$. From those two equation, note that $\tilde{x}^\ell$ increases when $\tilde{\delta}$ increases and/or y^ℓ increases and/or y^h-y^ℓ decreases. Now we can check when this sufficient condition holds. Suppose that $\tilde{\delta}=0$. (A.16) is satisfied when the aggregate risk, y^h-y^ℓ, is sufficiently large and agents are sufficiently risk averse with high γ and the supply of assets is sufficiently large with high level of y^ℓ. Note that when $\tilde{\delta}$ increases, the right side of (A.15) while the left side of (A.15) decreases since $\tilde{x}^\ell$ goes up and the gap between $\tilde{x}^h$ and $\tilde{x}^\ell$ shrinks. So, if (A.15) is satisfied at $\tilde{\delta}=0$, there is an optimal capital requirement, $\delta^{h*} \in (0,\bar{\delta})$, because (A.15) holds equality at the value greater than one by the intermediate value theorem and the left side of (A.15) is one at $\tilde{\delta}=\bar{\delta}$. QED

Lemma 3. *For (x^h, x^ℓ) on F curve, if $x^h > x^{h*}$ and $x^\ell < x^{\ell*}$, then the slope of F curve (39) is steeper than the slope of the welfare curve (13).*

Proof. *By using $-\frac{xu^{''}(x)}{u^{'}(x)} = \gamma$ we can re-write (41) as*

$$\frac{\partial x^\ell}{\partial x^h} \mid V = -\frac{\pi(1-\gamma+\gamma\frac{K_1^{'}(x^h,x^\ell)}{x^h u^{''}(x^h)})u^{'}(x^h)}{(1-\pi)(1-\gamma+\gamma\frac{K_2^{'}(x^h,x^\ell)}{x^\ell u^{''}(x^\ell)})u^{'}(x^\ell)} \gtreqless -\frac{\pi\{u^{'}(x^h)-1\}}{(1-\pi)\{u^{'}(x^\ell)-1\}} = \frac{\partial x^\ell}{\partial x^h} \mid W \quad \text{(A.17)}$$

where $K_1^{'}(x^h,x^\ell) = u^{''}(x^h)\frac{\beta\{y^h-\beta(y^h-y^\ell)(1-\pi)u^{'}(x^\ell)\}}{\{1-\beta\pi u^{'}(x^h)+\beta(1-\pi)u^{'}(x^\ell)\}^2}$ and $K_2^{'}(x^h,x^\ell) = u^{''}(x^\ell)\frac{\beta\{y^\ell+\beta(y^h-y^\ell)\pi u^{'}(x^h)\}}{\{1-\beta\pi u^{'}(x^h)+\beta(1-\pi)u^{'}(x^\ell)\}^2}$.

Note that $\frac{K_1'(x^h,x^\ell)}{u''(x^h)} > \frac{K_2'(x^h,x^\ell)}{u''(x^\ell)} > 0$ *because* $1-\beta\{\pi u'(x^h)+(1-\pi)u'(x^\ell)\} > 0$ *as long as* $\psi > 0$ *in equilibrium. So we can rearrange (A.17) as*

$$\left(1-\gamma+\gamma\frac{K_1'(x^h,x^\ell)}{x^h u''(x^h)}\right)\frac{u'(x^h)}{u'(x^h)-1} \gtreqless \left(1-\gamma+\gamma\frac{K_2'(x^h,x^\ell)}{x^\ell u''(x^\ell)}\right)\frac{u'(x^\ell)}{u'(x^\ell)-1}, \tag{A.18}$$

where the equality holds at $(x^{h*}, x^{\ell*})$. *When* (x^h, x^ℓ) *moves from the point* CE *to the point* A *in Figure 6,* $\frac{u'(x^h)}{u'(x^h)-1}$ *increases and* $\frac{u'(x^\ell)}{u'(x^\ell)-1}$ *decreases. Thus, if* $\gamma = 0$ *then the left-hand side of (A.18) is greater than the right-hand side: The slope of* F *curve is steeper than the slope of the welfare curve. This inequality still holds even though* γ *approaches to one. If* $\gamma = 1$ *then (A.18) can be reduced into*

$$\begin{aligned}&(y^\ell+\beta(y^h-y^\ell)\pi u'(x^h))\frac{x^h}{u'(x^h)} - (y^h-\beta(y^h-y^\ell)(1-\pi)u'(x^\ell))\frac{x^\ell}{u'(x^\ell)}\\ \gtreqless&(y^\ell+\beta(y^h-y^\ell)\pi u'(x^h))x^h - (y^h-\beta(y^h-y^\ell)(1-\pi)u'(x^\ell))x^\ell.\end{aligned} \tag{A.19}$$

Note that the left-hand side of (A.19) is greater than the right-hand side when x^h *increases and* x^ℓ *decreases because* $\frac{x^h}{u'(x^h)} - \frac{x^\ell}{u'(x^\ell)}$ *becomes greater than* $x^h - x^\ell$. *QED*

Proposition 4. *In region 2 and 3 when the state-contingent monetary policy is limited with small* V, *the state-contingent capital requirements can improve the welfare of the equilibrium allocation further if* γ *is sufficiently large and/or the gap between* y^h *and* y^ℓ *is sufficiently large.*

Proof. *Suppose that we have an equilibrium allocation* $(\bar{x}^h, \bar{x}^\ell, \bar{\psi}, \bar{\mu}^\ell)$ *at* $\delta^h = \delta^\ell = 0$ *and* $R = 0$ *in which* $\bar{x}^h > \bar{x}^\ell$ *holds in region 2 or 3. We can follow the proofs of Lemma 1, Proposition 2 and 3 by replacing the equilibrium condition for the state* ℓ *(15) with*

$$\frac{V}{(1-\pi)u'(x^\ell)} + \beta(\psi + y^\ell) = x^\ell$$

which comes from (30) and (33). Then the partial derivative, $\frac{\partial x^\ell(\psi)}{\partial \psi} \mid_{\delta^h=0}$, *will be greater than* β *in (A.1) as*

$$\frac{\partial x^\ell(\psi)}{\partial \psi} \mid_{\delta^h=0} = \frac{\beta}{1+\frac{Vu''(x^\ell)}{(1-\pi)u'(x^\ell)}} > \beta.$$

Thus the proofs of Lemma 1, Proposition 2 and 3 will be applied straight-forward: In region 2 capital requirements can improve the welfare at $\delta^h = \tilde{\delta}$. *In region 3 if* $\frac{\beta(y^h-y^\ell)}{x^*} > \{1 + \frac{(1-\beta)}{(\beta-\beta\pi)\gamma}\}^{\frac{1}{\gamma}} - 1$ *holds, then capital requirements can improve the welfare at* $\delta^h = 0$. *QED*