Scarcity of Assets, Private Information, and the Liquidity Trap *

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Abstract

This paper explores how scarcity of assets and private information can restrict liquidity insurance and the effectiveness of monetary policy. A micro-founded money-search model is constructed in which agents use liquid assets for consumption. Given an idiosyncratic liquidity risk, a banking contract is considered for providing liquidity insurance between patient and impatient agents. However, under lack of record-keeping technology it is difficult to reveal the types by liquidity needs. When the supply of assets is not sufficiently large, illiquid assets for patient agents are useful for revealing the types. Therefore, there exists a liquidity premium on illiquid assets although illiquid assets are not used directly in exchange. Given the scarcity of the assets, a proportion of liquid assets could be held in balance sheets of banks, i.e. excess reserves, for providing to patient agents. With these excess reserves, a liquidity trap, in which exchanging liquid assets with illiquid assets is no longer effective, can occur away from the zero lower bound. **Key Words**: *liquidity insurance, truth-telling constraints, separating equilibrium, excess reserves* **JEL Codes**: E42,E44,E52.

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1 Introduction

Liquidity trap, in which monetary policy is no longer effective, has been a subject of interest to both monetary theorists and central bankers for a long time. Given its definition liquidity trap is associated with excess reserves because exchanging reserves with government bonds is no more effective in the real macroeconomic variables. One conventional explanation for excess reserves is based on lack of good loan opportunities. For example, in recessions commercial banks would hold excess reserves voluntarily since the expected return of projects is lower than the rate of return in reserves. In the Great Depression of the 1930s excess reserves in the banks have increased with gold inflow, while bank credit failed to expand. The other widespread view for excess reserves is related to the zero lower bound. When the central bank lowers the level of nominal interest rates to zero, there is no more demand for government bonds since money or reserves can replace the role of government bonds. However, this view has been tested during the recent Great Recession. After the Lehman Brothers went bankrupt on September 2008, the federal fund rate fluctuates under the nominal interest target of the central bank and deposit institutions hold excessive reserves, even though there is no interest on reserves yet as shown in Figure 1. In this paper I develop a new theory of liquidity trap in which excess reserves are useful for providing liquidity insurance by separating the types under private information.

[Figure 1 here]

Providing liquidity insurance is one of the primary functions of banks. When people are exposed to idiosyncratic liquidity risk, separating the types by liquidity needs *ex post* is beneficial for efficient liquidity distribution. If liquid assets are able to facilitate transactions for consumptions, two different liquid assets could be useful to separate the types under private information. For example, a bank can provide liquid assets to high marginal utility agents as promising less liquid assets to low marginal utility agents. The low marginal utility types have an incentive to mimic high marginal utility types, but if the illiquid assets for the low types are plentiful then we can prevent the low types from mimicking the high types. However, if the illiquid assets are scarce, then the bank is required to hold additional liquid assets for the low marginal utility types in order to separate the types. In this respect excess reserves can exist in the balance sheet of banks to discourage the low marginal utility types from withdrawing liquid assets. In this case exchanging illiquid assets with liquid assets is no longer effective because the bank will still hold newly injected liquid assets in the balance sheet instead of illiquid assets.

In order to explore this issue I modify Lagos and Wright (2005), specifically Rocheteau and Wright (2005), type asset-exchange model in which *ex ante* heterogeneous agents trade in decentralized meetings and rebalance their portfolio in a centralized market. This microfounded model is useful to incorporate informational frictions such as lack of record-keeping and limited commitment because those informational frictions are also essential for the existence of the assets. For example, one important assumption of this paper is lack of record-keeping technology. This anonymity assumption makes not only liquid assets essential for decentralized trade, but also inhibits the bank in revealing the types under private information.[1] Thus, there arises a need to separate types by using liquid and illiquid assets naturally. Moreover, this quasi-linear model is highly tractable with multiple assets and banking contracts. In the baseline model I use only two different liquid Lucas trees to show the main intuition that illiquid assets can be useful for revealing private information given the scarcity of assets. In the extended model I introduce money and government bonds additionally in order to consider the effectiveness of monetary policy, specifically open-market-operations. In this setting we can learn more about the relationship between the liquidity trap and the zero nominal interest rates.

This paper provides several key findings. First, when the total supply of assets is insufficient to separate the types, a liquidity premium could arise in the price of illiquid assets even though illiquid assets are not useful for trade directly at all. A liquidity premium on the illiquid assets arises because illiquid assets are useful for revealing the types as well as liquid assets under private information. Thus when the supply of assets is scarce to separate the types, both liquid and illiquid assets will have a liquidity premium and the liquidity premia are the same because both assets are indifferent for low marginal utility agents.

Second, in the model when the supply of assets is insufficient to separate the types, a proportion of liquid assets, i.e. excess reserves, must be held in the balance sheet of the bank for the low marginal utility types. In this case when the government injects money, i.e. liquid assets, in the markets by absorbing government bonds, i.e. illiquid assets, the sum of excess reserves and government bonds held in the bank will not change at all because the bank requires to hold the same amount of assets for low utility types. Thus liquidity trap equilibrium can exist when the truth-telling constraint binds in the model and this liquidity trap is not generated by zero nominal interest rates.

Lastly, this paper shows a relationship between excess reserves and bank runs explicitly. In the model a run-proof banking contract is considered with truth-telling constraints for liquidity insurance. Therefore, when the supply of illiquid assets is scarce, the excess reserves are held by banks because it is useful to avoid bank runs.

1.1 Related Literature

This paper is related to the literature that studies a liquidity premium in asset prices. Geromichalos et. al. (2007) and Lagos and Rocheteau (2008) show that there is a liquidity premium on the asset prices when the assets are useful for transactions by using the Lagos-Wright framework. Herrenbrueck and Geromichalos (2016) also show that the price of illiquid assets has a liquidity premium if the illiquid assets can be traded to obtain liquid assets. However, this paper is different from those papers because illiquid assets are useful for revealing private information instead of being used for exchange or traded for liquidity.

There is a literature in which the illiquid assets or the legal restriction on government bonds are socially beneficial. Kocherlakota (2003) shows that illiquid assets are beneficial because agents can trade liquid and illiquid assets after observing idiosyncratic shock. Shi (2008) shows that legal restriction on government bonds improves welfare when low marginal utility type cannot trade with bonds. However, this paper does not claim that illiquid assets

[1] If record-keeping is available, credit or (proportional) tax scheme can replicate the optimal equilibrium allocation even under private information.

are beneficial, because liquid assets can also be used by low marginal utility agents in the model.

The liquidity trap result is related to the literature on the implementation of monetary policy. Wallace (1981) is the seminal paper that studies on the effectiveness of monetary policy, in particular open market operations. In his model open-market-operations are irrelevant unless money and government bonds are different in a cost of transaction, i.e. liquidity. Thus a return dominance between rates of return on money and government bonds seems to be necessary for the effectiveness of monetary policy. In this respect the Friedman rule equilibrium is a liquidity trap because there is no friction in the money trade so that the rate of return on money is the same as the rate of return on government bonds. Recently, Williamson (2012) shows that a liquidity trap equilibrium can exist away from the Friedman rule when the assets are scarce in an economy. In his paper the rates of return on money and government bonds can also be equal because the liquidity premium on both assets are the same although there exist trade frictions by the scarcity of assets. In this paper a liquidity trap exists because of excess reserves generated by private information. Thus in equilibrium the rates of return on money and government bonds are the same, but it is just a result of excess reserves. Rocheteau, Wright, and Xiao (2015) study open market operations with various specifications. They shows that a liquidity trap can exist when the supply of government bonds is sufficiently small, given that one type of agents trade only when money while the other type of agents trade with money and/or government bonds. This paper can confirm their result and provide a specific reasons for this phenomenon.

This paper is also related to a literature on liquidity insurance. Diamond and Dybvig (1983) shows that bank runs can exist as an equilibrium outcome when liquidity shocks are private information in their pioneering paper. Allen and Gale (1998) point out that the rate of return on long-term projects are critical to reveal the private information in Diamond-Dybvig model: If the rate of return on long-term project is high enough then truth-telling constraint does not bind. In this paper the key feature for preventing bank run is the sufficient supply of illiquid assets rather than the high rate of return on long-term projects because this model uses assets to overcome trade frictions explicitly.

There are some banking models with an explicit monetary trade. Freeman (1988) and Champ, Smith, and Williamson (1996) studies banking and liquidity insurance with overlapping generation models. Recently, Bencivenga and Camera (2011) studies insurance banking in Lagos and Wright (2005) framework, but a standard debt contract is considered. This paper builds on Williamson (2012), in which a Diamond-Dybvig type bank provides liquidity insurance, and add individual incentive constraints to study private information friction.

2 Model

The basic model structure is based on Rocheteau and Wright (2005) in which *ex ante* heterogeneous agents trade in bilateral meetings and rebalance their portfolio in the centralized competitive market. Time $t = 0, 1, 2, \ldots$ is discrete in infinite horizon and each period is divided into two sub-periods - the Centralized Meeting(CM) followed by the Decentralized Meeting(DM). There is a continuum of buyers and sellers, each with unit mass. An individual buyer has preferences

$$E_0\sum_{t=0}^{\infty}\beta^t[-H_t+\theta_t^i u(x_t)]$$

where $H_t \in \mathbb{R}$ is labor supply of the buyer in the *CM*, $x_t \in \mathbb{R}_+$ is consumption of the buyer in the *DM*, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u^{'}(0)=\infty$, $u(0)=0$, and $-x\frac{u^{''}(x)}{u^{'}(x)} < 1$ for all $x > 0$.[2] One variation is that each buyer is exposed to an idiosyncratic liquidity shock by type i, θ_t^i, which follows independent and identical distribution with two realizations $\{1,\theta\}$ where $0 \le \theta < 1$. A buyer becomes high marginal utility type 1, $\theta_t^1 = 1$, with probability $\rho \in (0,1)$ and otherwise the buyer becomes low marginal utility type 2, $\theta_t^2 = \theta$. Note that there is *ex post* heterogeneity in marginal utility across the types.[3] Each seller has preferences as

$$E_0\sum_{t=0}^{\infty}\beta^t[X_t-h_t]$$

where $X_t \in \mathbb{R}$ is consumption of the seller in the *CM*, and $h_t \in \mathbb{R}_+$ is labor supply of the seller in the *DM*. All the agents can consume and produce in the *CM*. In the *DM* buyers can consume, but cannot produce while sellers can produce, but cannot consume. One unit of labor inputs can produce one unit of perishable consumption goods in the *CM* or in the *DM*.

In the beginning of the *CM*, all agents meet together and previous debts are settled, if they were. Buyers receive lump-sum transfer or pay lump-sum tax and the holders of the Lucas trees receive the realized dividends. A Walrasian market opens, assets and goods are traded competitively. In the *DM* each buyer meets each seller bilaterally and the terms of trade are determined by bargaining. I assume that the buyer make a take-it-or-leave-it offer to the seller in the meetings for simplicity. I also assume that all agents are anonymous and there is no public record-keeping technology in the*DM*. Thus recognizable assets are essential for trade in the*DM* and all trade must be*quid pro quo*.

There are two types of divisible Lucas trees. One tree, named as liquid tree, is useful for both types of buyers' trade while the other tree, named as illiquid tree, is only useful for type 2 buyers' trade. Liquid and illiquid trees are endowed to buyers in the initial period *CM* with fixed supply α and $1-\alpha$, respectively, and pays the same dividend y in every period *CM*. In period t *CM* liquid tree trades at the price ψ_t^l in terms of goods while illiquid tree trades at the price ψ_t^i.[4]

Given idiosyncratic liquidity shock a banking arrangement arises endogenously to allocate two different liquid assets by the types of buyers efficiently. Without a banking contract, type 1 buyers could hold useless illiquid assets and type 2 buyers could hold liquid assets

[2] In the model asset demand is strictly increasing in rates of return when the coefficient of relative risk aversion is less than one. It guarantees to have at least one equilibrium exists.

[3] This heterogeneity in marginal utility is different from Shi (2008). In his model bonds can be beneficial because bonds are used only for higher marginal utility trade while money is used for all trade. In this paper bonds or illiquid assets cannot be used in type 1 trade, but marginal utility of type 1 trade is greater than marginal utility of type 2 trade. So illiquid assets do not have any advantage in trade.

[4] This Lucas tree represents private investment. Since the supply is fixed, the inefficiency is reflected to its price instead of the quantity.

instead. Thus it is socially optimal to have a banking contract that provides liquid assets to type 1 buyers and illiquid assets to type 2 buyers. In the model all agents can propose a banking contract for liquidity insurance and can become a bank. Banks observe the size of liquidity shock ρ exactly, but cannot verify the type of individual buyers under private information. So there is a possibility that one type of buyers can mimic the other type of buyers. In the model type 1 buyers do not have an incentive to mimic since illiquid assets are useless for them. But, type 2 buyers can mimic type 1 buyers so that banks need to hold a sufficient amount of liquid or illiquid assets for type 2 buyers to prevent bank runs. I assume that illiquid assets cannot be liquidated so that there is no bank-run such as shown in Diamond and Dybvig (1983) even though type 2 buyers mimic type 1 buyers. To support banking arrangement with stability I assume that buyers can meet only the bank after their liquidity shock is realized. If *ex post* asset-trading among buyers is allowed then banking contract is unraveled and collapsed as shown in Jacklin (1987).[5]

Timing is as follows. In the beginning of *CM* all buyers provide labor and trade liquid and illiquid trees with sellers in a Walrasian market. Buyers deposit either goods or trees to a banker with a banking contract. After liquidity shock is realized, buyers learn their types and ρ buyers meet the banker to withdraw liquid trees. In the *DM* buyers meet sellers randomly in the bilateral meeting and make take-it-or-leave-it offers.

[Figure 2 here]

3 Competitive Equilibrium

In this section to emphasize private information I assume that illiquid assets are not useful for trade at all with $\theta = 0$. In the following I consider each perfect information and private information case and compare the equilibrium allocations to understand the reason why the liquidity insurance can be restricted by the private information. Under private information banks suggest two type-dependent consumption offers $\{(x_1^l, x_1^i), (x_2^l, x_2^i)\}$ for type 1 and type 2 buyers to reveal their types. Note that superscripts denote type of assets between *liquid* and *illiquid* while subscripts denote type j of buyers: x_{jt}^l, x_{jt}^i represents the consumption level of type j buyers with liquid and illiquid assets. Under perfect competition a representative bank suggests a banking contract to maximize buyers' ex ante expected value. In equilibrium the bank solves the following generalized problem in the *CM* of period t:

$$\underset{d_t, a_t^l, a_t^i, x_{1t}^l, x_{1t}^i, x_{2t}^l, x_{2t}^i}{Max} - d_t + \rho\{u(x_{1t}^l) + x_{1t}^i\} + (1-\rho)\{x_{2t}^l + x_{2t}^i\} \tag{1}$$

subject to a participation constraint of the bank,

$$d_t - \psi_t^l a_t^l - \psi_t^i a_t^i + \{\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l - (1-\rho)x_{2t}^l\} + \{\beta(\psi_{t+1}^i + y^i)a_t^i - \rho x_{1t}^i - (1-\rho)x_{2t}^i\} \geq 0 \tag{2}$$

[5] Note that a banking contract equilibrium provides higher welfare than assets-trading market equilibrium. It is because a bank contract can provide resources to high marginal utility agents as much as it can regardless of prices.

and incentive constraints of the bank,

$$\beta(\psi^l_{t+1} + y^l)a^l_t - \rho x^l_{1t} - (1-\rho)x^l_{2t} \geq 0 \tag{3}$$

$$\beta(\psi^l_{t+1} + y^l)a^l_t + \beta(\psi^i_{t+1} + y^i)a^i_t - \rho\{x^l_{1t} + x^i_{1t}\} - (1-\rho)\{x^l_{2t} + x^i_{2t}\} \geq 0 \tag{4}$$

and truth-telling constraints,

$$u(x^l_{1t}) + x^i_{1t} \geq u(x^l_{2t}) + x^i_{2t} \tag{5}$$

$$x^l_{2t} + x^i_{2t} \geq x^l_{1t} + x^i_{1t} \tag{6}$$

and non-negative constraints,

$$d_t, a^l_t, a^i_t, x^l_{1t}, x^i_{1t}, x^l_{2t}, x^i_{2t} \geq 0 \tag{7}$$

All quantities in equations (1)-(7) are expressed in units of the *CM* good in time t. The problem (1) subject to constraints (2)-(7) states that a banking contract is chosen in equilibrium to maximize the expected utility of the representative buyer subject to the participation constraint for the banks (2) and liquid asset constraint (3) and collateral constraint (4) and individual incentive constraints by types (5)-(6) and non-negativity constraints (7). In (1)-(7) d_t denotes deposit of buyers, a^l_t, a^i_t denote demand for liquid and illiquid asset holdings of banks, ψ^l_t, ψ^i_t denote the prices of liquid and illiquid assets in the *CM*, respectively. The quantity on the left side of (2) is the net payoff for banks. In the *CM* of time t the banks receive d_t deposits and invest in liquid and illiquid assets with market prices then the banks pay x^l_{jt} to each type buyer before the *DM* and pay x^i_{jt} to the holders of deposit claims in the following *CM*. The participation constraint (2) implies that when deposit claims are paid off, the net payoff for the banks must be greater than zero. The liquid asset constraint (3) implies that type 1 buyers can withdraw liquid assets by the limit of liquid asset holdings. The collateral constraint (4) implies that the liquid and illiquid assets can be seized when the bank decides to abscond. Individual incentive constraint (5)-(6) represent that each type of buyer weakly prefer an offer for own type to the offer for other type after type shock is realized. Note that type 1 buyers can also consume with illiquid assets in the next *CM*.

3.1 Perfect Information

For a benchmark I consider competitive equilibrium with perfect information. In case of perfect information banks know the buyer's type exactly after the liquidity shock is realized. In equilibrium *ex post* banks will provide all of liquid assets to type 1 buyers who only can trade in the *DM*. Since illiquid assets are useless for trade banks will not hold these assets as long as the real return of the illiquid asset are less than time preference. If the real return of the illiquid asset is same as time preference then banks can hold these illiquid assets and provide them to type 1 or 2 buyers, but it is irrelevant since both type buyers have linear utility function for illiquid assets. Thus without loss of generality I assume that banks do not hole illiquid assets, i.e. $x^i_{1t} = x^i_{2t} = 0$, with perfect information. Moreover, truth-telling constraints are unnecessary. Thus given price ψ^l_t, a representative bank solves the reduced maximization problem in the *CM* of period t:

$$\underset{d_t, a_t^l, x_{1t}^l}{Max} \; - d_t + \rho u(x_{1t}^l) \tag{8}$$

subject to the participation constraint,

$$d_t - \psi_t^l a_t^l + \{\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l\} \geq 0 \tag{9}$$

and liquid asset constraint,

$$\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l \geq 0 \tag{10}$$

and non-negative constraints,

$$d_t, a_t^l, x_{1t}^l \geq 0 \tag{11}$$

By plugging (9) into (8) we have the first-order conditions by a_t^l, x_{1t}^l,

$$\psi_t^l = \beta(\psi_{t+1}^l + y^l)(1 + \lambda_t) \tag{12}$$

$$u^{'}(x_{1t}^l) - 1 = \lambda_t \tag{13}$$

where λ_t is a multiplier associated with liquid asset constraint (10). The first-order conditions (12) and (13) can be reduced to

$$\psi_t^l = \beta(\psi_{t+1}^l + y^l)u^{'}(x_{1t}^l) \tag{14}$$

In equilibrium asset market clear in the *CM* and a representative bank holds all the liquid asset in its portfolio for $t = 0, 1, 2, \ldots$. The supply of liquid asset is equal to the demand of banks as

$$a_t^l = 1. \tag{15}$$

Definition 1. *Given (ρ, y^l, y^i) a stationary competitive equilibrium under perfect information consists of quantity x_{1t}^l and price ψ_t^l and multiplier λ which solves equations (10), (14), (15).*

In what follows I focus on stationary equilibrium allocations without time scripts on variables. There are two equilibrium cases following by the value of y^l.

Case (i) Suppose that the liquid asset constraint (10) does not bind. That means, in equilibrium the first-best consumption level, x^* where $u^{'}(x^*) = 1$, is achieved for type 1 buyers. From the first-order condition (14), the asset price is the same as its fundamental value:$\psi^l = \psi_f^l$ holds where $\psi_f^l := \frac{\beta y^l}{1-\beta}$. Note that this case of equilibrium is supported by $y^l \geq \frac{(1-\beta)}{\beta}\rho x^*$ from (10).

Case (ii) Suppose that the liquid asset constraint (10) binds with $y^l < \frac{(1-\beta)}{\beta}\rho x^*$. Then the equilibrium allocation (x_1^l, ψ^l) is uniquely determined from (10) and (14) since ψ^l is strictly increasing in x_1^l from (10) while ψ^l is strictly decreasing in x_1^l from (14). Note that the consumption level is less than its optimal level, $x_1^l < x^*$ and the asset price is greater than its fundamental value,$\psi^l > \psi_f^l$ in equilibrium. Liquidity premium, the difference between the asset price and its fundamental value is strictly positive because of liquid asset shortage.

The price of illiquid asset is same as its fundamental value as $\psi^i = \psi^i_f$ where $\psi^i_f := \frac{\beta y^i}{1-\beta}$, if it is traded in the market.

These two cases can be described in Figure 3. If the supply of liquid asset is large enough with $y^l \geq \frac{(1-\beta)}{\beta}\rho x^*$ then we have the case 1 equilibrium with the first-best allocation. If the supply of liquid asset is low with $y^l < \frac{(1-\beta)}{\beta}\rho x^*$ then we would have this case 2 equilibrium.

[Figure 3 here]

3.2 Private Information

In case of private information as described above, banks solve the original maximization problem (1)-(7). To simplify the problem I use some lemmas here.

Lemma 1. *(Single Crossing Property) In equilibrium with $x^l_{1t} \in [0, x^*)$, both truth-telling constraints do not bind simultaneously.*

Proof. *If both truth-telling constraints (5) and (6) bind then $u(x^l_{1t}) - x^l_{1t} = u(x^l_{2t}) - x^l_{2t}$. Since $u(x) - x$ is strictly increasing in $x \in [0, x^*]$, $x^l_{1t} = x^l_{2t}$ and $x^i_{1t} = x^i_{2t}$. Note that $x^i_{1t} = x^i_{2t} > 0$ for $y > 0$ in equilibrium. For $x^l_{1t} = x^l_{2t} < x^*$the expected value of buyers can increase by transferring liquid assets from type 2 buyers to type 1 buyers and transferring the same amount of illiquid assets from type 1 buyers to type 2 buyers. Contradiction. QED.*

Lemma 2. *In equilibrium with $x^l_{1t} \in [0, x^*)$, the truth-telling constraint for type 1 buyers does not bind.*

Proof. *Suppose that the constraint (5) binds while the constraint (6) does not bind. If $x^l_{2t} > 0$ then the expected value of buyers can increase by transferring liquid assets from type 2 buyers to type 1 buyers. If $x^l_{2t} = 0$ then the expected value of buyers are indifferent when illiquid assets are transferred from type 2 buyers to type 1 buyers that means (6) does not bind. Contradiction. QED.*

In equilibrium the truth-telling constraint for type 1 buyers (5) does not bind. It is because banks allocate resources to type 1 buyers who have higher marginal utility as much as possible to maximize the expected utility for agents. Thus the incentive constraint for type 2 buyers always binds. In Figure 4, indifference curve of type 1 buyers intersects the indifference curve of type 2 buyers at x^l_{1t}. To keep utility for type 2 buyers x^i_{2t} is required. Since (5) does not bind, there is no difference between x^i_{2t} and x^l_{2t} in the problem so that they can be merged as x_{2t}. Note that type-dependent contract is non-linear in general, but in this case deposit contract is linear as standard deposit contract because quasi-linear utility is adopted.

[Figure 4 here]

Lemma 3. *In equilibrium with $x^l_{1t} \in [0, x^*)$,$x^i_{1t} = 0$.*

Proof. *Suppose that $x^i_{1t} > 0$ in equilibrium. If the truth-telling constraint for type 2 buyers (6) binds then transferring illiquid assets to type 2 buyers can relax (6). If (6) does not bind then the expected value of buyers is indifferent. Thus there is no reason to have $x^i_{1t} > 0$ in equilibrium. QED.*

Illiquid assets for type 1 buyers are unnecessary because neither it is used for trade nor it overcomes private information friction in (12). Since $x^i_{1t} = 0$ in equilibrium we can rename x^l_{1t} as x_{1t}. Without the truth-telling constraint for type 1 buyers (5) and with choice variable $x_{1t}$and x_{2t}, we have the first-order conditions by $a^l_t, a^i_t, x_{1t}, x_{2t}$,

$$\psi^l_t = \beta(\psi^l_{t+1} + y^l)(1 + \lambda_{1t} + \lambda_{2t}) \tag{16}$$

$$\psi^i_t = \beta(\psi^i_{t+1} + y^i)(1 + \lambda_{2t}) \tag{17}$$

$$\rho u^{'}(x_{1t}) - \rho - \rho\lambda_{1t} - \lambda_{3t} = 0 \tag{18}$$

$$(1 - \rho) - (1 - \rho) - (1 - \rho)\lambda_{2t} + \lambda_{3t} = 0 \tag{19}$$

where λ_{1t},λ_{2t} and λ_{3t} denote each multiplier associated with the constraints (3), (4) and (6). In equilibrium asset markets clear in the *CM* and a representative bank holds all the liquid and illiquid assets in its portfolio for $t = 0, 1, 2, \ldots$. The supply of liquid and illiquid asset is equal to its demand from banks, respectively, as shown in (20).

$$a^l_t = a^i_t = 1 \tag{20}$$

Definition 2. *Given (ρ, y^l, y^i) a stationary competitive equilibrium under private information consists of quantity x_1,x_2 and price ψ^l_t,ψ^i_t and multiplier λ_{1t},λ_{2t},λ_{3t} which solves equations (3)-(4),(6),(16)-(20).*

I focus on stationary equilibrium allocations without time scripts on variables. Note that the collateral constraint (4) and the truth-telling constraint (6) either binds or does not bind together from (19) because $x_2 > 0$ in equilibrium with $y^i > 0$. If the truth-telling constraint (6) binds with $\lambda_3 > 0$ then the liquid asset constraint (3) is relaxed with $\lambda_1 = 0$ because only one of them can restrict $x_1 \in [0, x^*)$ in equilibrium. In sum under private information there are three equilibrium cases. As discussed in perfect information subsection we have case (i) and (ii) equilibrium and additionally a new equilibrium, case (iii) equilibrium, in which the truth-telling constraint (6) binds and the liquid asset constraint (3) is relaxed.

Case (i) When the constraints (3), (4), (6) do not bind we have

$$\frac{\psi^l}{\beta(\psi^l + y^l)} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \tag{21}$$

and

$$1 = u^{'}(x_1) \tag{22}$$

from the first-order conditions (16)-(18). Since the constraints do not bind, the optimal level of consumption is achieved, $x_1 = x^*$, for type 1 buyers and the asset prices are the same as their fundamental values, $\psi^l = \psi^l_f$, $\psi^i = \psi^i_f$ from (21)-(22). The equilibrium is supported by a region with $\psi^l_f \geq \rho x^*$ and $\psi^l_f + \psi^i_f \geq x^*$ from (3)-(4), (20). Note that this first-best equilibrium allocation is the same as one in perfect information case (i). One interesting point is that this first-best equilibrium allocation is feasible even when there exists a degree of private information as long as liquid and illiquid assets are plentiful in the economy.

Case (ii) When the liquid asset constraint (3) binds only, we have

$$\frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \tag{23}$$

and

$$\frac{\psi^l}{\beta(\psi^l + y^l)} = u^{'}(x_1) \tag{24}$$

from (16)-(18). The binding constraint (3) with asset market clear condition (20) can be reduced into

$$\beta(\psi^l + y^l) = \rho x_1 \tag{25}$$

Then (x_1, ψ^l) are uniquely determined from (24) and (25). In equilibrium we have $x_1 < x^*$,$\psi^l > \psi^l_f$, $\psi^i = \psi^i_f$ and $x_1 \leq x_2$. Note that a liquidity premium arises in the price of the liquid asset since the inefficiency is caused by the scarcity of liquid asset as described in Champ, Smith and Williamson (1996). On the other hand, the price of illiquid asset keeps at its fundamental value because those illiquid assets are already plentiful. The equilibrium is supported by a region which satisfies with $\psi^l_f < \rho x^*$ and $y^l \leq \frac{\rho}{1-\rho} y^i$. If y^i is too low then there exists a threshold point in which the truth-telling constraint (6) starts to bind while the liquid asset constraint (3) is just relaxed. In this threshold point we have $\beta(\psi^l + y^l) = \rho x_1$, $\beta(\psi^i + y^i) = (1-\rho)x_2$, $x_2 = x_1$ from (3), (4), (6). We also have $\frac{\psi^i}{\beta(\psi^i+y^i)} = \frac{\psi^l}{\beta(\psi^l+y^l)}$ from (16)-(17) since the liquid asset constraint (3) is just slack at the point. Note that those conditions can be reduced into $y^l = \frac{\rho}{1-\rho} y^i$ which is a threshold borderline between case (ii) and case (iii) equilibrium. Note that this case (ii) equilibrium is also the same as the case 2 equilibrium under perfect information.

Case (iii) When the collateral constraint (4) and the truth-telling constraint (6) bind, we have

$$\frac{\psi^l}{\beta(\psi^l + y^l)} = \frac{\psi^i}{\beta(\psi^i + y^i)} = \frac{\{\rho u^{'}(x) + 1 - \rho\} - \rho}{1 - \rho} \tag{26}$$

and

$$\beta(\psi^l + y^l) + \beta(\psi^i + y^i) = x \tag{27}$$

from (16)-(20) where $x := x_1 = x_2$ is denoted. Then (x, ψ^l, ψ^i) are uniquely determined from the equilibrium condition (26)-(27). In equilibrium we have $x < x^*$,$\psi^l > \psi^l_f$, $\psi^i > \psi^i_f$. Note that a liquidity premium arises in the both prices of liquid and illiquid assets although the liquid asset constraint does not bind in equilibrium. It is because both liquid and illiquid assets are scarce to reveal the private information in equilibrium. In equilibrium a proportion of liquid assets must be provided to type 2 buyers so that rates of return in liquid asset and illiquid asset are same in equilibrium. Note that a liquidity premium arises although illiquid assets are not useful for trade at all.

Proposition 1. *If $\frac{\beta y^l}{1-\beta} \geq \rho x^*$ and $\frac{\beta(y^l+y^l)}{1-\beta} < x^*$ then $\psi^i > \psi^i_f$ in equilibrium.*

Three regions of equilibrium under private information are described in Figure 5. Notice that the first-best equilibrium allocation appears when liquid assets are plentiful and are supported by sufficient illiquid assets. The case (ii) equilibrium allocation is shown when liquid assets are scarce although those liquid assets are supported by illiquid assets enough. The case (iii) equilibrium allocation arises when the types are hardly revealed because of the scarcity of illiquid assets.

[Figure 5 here]

4 Monetary Equilibrium

In this section I introduce two types of nominal government-issued assets in order to consider how this private information restricts the implementation of monetary policy. Fiat money trades at price ϕ_t in terms of goods in the competitive market of period t *CM*. One-period maturity government bonds is an obligation to pay one unit of fiat money in the period $t+1$ *CM*. The price of government bonds is z_t in terms of goods in the period t *CM*. In this extended model government consists of fiscal authority and monetary authority. Fiscal authority can enforce lump-sum tax or provide transfer to buyers in the *CM* and issue government bonds and pay interests in the next *CM*. Monetary authority can issue fiat money and also inject or absorb fiat money in the asset market by exchanging money with government bonds, i.e. open-market-operations(OMOs). I assume that private assets are not eligible to be an object for OMOs. At $t = 0$ government bonds are issued and fiat money is injected by OMOs and then the revenue of issuing government bonds and fiat money is transferred to buyers. After $t = 0$ outstanding fiat money and government bonds can be supported by tax or transfer over time. So the consolidated government budget constraints are described as

$$\phi_0(M_0 + z_0 B_0) = \tau_0 = V$$

and

$$\phi_t\{M_t - M_{t-1} + z_t B_t - B_{t-1}\} = \tau_t,\ t = 1, 2, 3, \dots$$

where M_t and B_t denote the nominal quantities of fiat money and government bonds held by private sector in the *CM* at time t, respectively. τ_t denote the real value of the lump-sum transfer from each buyer to the fiscal authority in the *CM* at period t. I assume that the fiscal authority keeps the total value of the outstanding consolidated government debt as a constant V after it is transferred with an exogenously fixed amount at $t = 0$. Thus in every period to maintain the real value of outstanding consolidated government debt, the real term of lump-sum transfer τ_t is derived passively from

$$\tau_t = \underbrace{(V_t - \frac{\phi_t}{\phi_{t-1}} V_{t-1})}_{seigniorage} + \underbrace{\frac{\phi_t}{\phi_{t-1}}(1 - z_{t-1})\phi_{t-1} B_{t-1}}_{real\,interest\,payment},\ t = 1, 2, 3, \dots,$$

Note that the lump-sum transfer consists of seigniorage from inflation and real interest payment for government bonds. The fixed real value of consolidated government debt assumption allows us to concentrate on monetary policy without the effect of fiscal policy.

Moreover, when V is assumed as small enough, assets can be insufficient to support the optimal level of consumption. In order to focus on OMOs, I assume that there are no private assets such as Lucas trees in this extended model with $y^l = y^i = 0$.

In this extension I also assume that the marginal utility of type 2 buyers is strictly positive with $\theta \in (0, 1)$. Thus type 2 buyers can consume with money and government bonds, but the marginal utility is lower than the type 1 buyer's.[6] I still assume $\theta < 1$ because I focus on a situation where the truth-telling constraint binds under private information.[7]

Since I add government debts, time line is changed. In the beginning of *CM* government-bonds holders can receive a unit of fiat money by redeeming a unit of government bonds. Then buyers receive lump-sum transfer(or pay lump-sum tax). Then all buyers provide labor and trade assets with sellers in a Walrasian market. Buyers deposit goods into a bank with a banking contract. After liquidity shock is realized, buyers learn their types and ρ buyers meet the banker to withdraw money and $1 - \rho$ buyers withdraw money and government bonds. In the *DM* buyers meet sellers randomly in the bilateral meeting and make take-it-or-leave-it offers.

Given perfect competition, a representative banking contract solves the following modified problem in the *CM* of period t:

$$\underset{d_t, m_t, b_t, a_t, x_{1t}, x_{2t}}{Max} - d_t + \rho u(x_{1t}) + (1 - \rho)\theta u(x_{2t}) \tag{28}$$

subject to the participation constraint,

$$d_t - m_t - z_t b_t - \psi_t^i a_t^i + \{\frac{\beta\phi_{t+1}}{\phi_t} m_t - \rho x_{1t}\} + \{\frac{\beta\phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1}^i + y^i)a_t^i - (1 - \rho)x_{2t}\} \geq 0 \tag{29}$$

and the cash constraint,

$$\frac{\beta\phi_{t+1}}{\phi_t} m_t - \rho x_{1t} \geq 0 \tag{30}$$

and the collateral constraint,

$$\frac{\beta\phi_{t+1}}{\phi_t}(m_t + b_t) + \beta(\psi_{t+1}^i + y^i)a_t^i - \rho x_{1t} - (1 - \rho)x_{2t} \geq 0 \tag{31}$$

and the truth-telling constraint,

$$x_{2t} \geq x_{1t} \tag{32}$$

and non-negative constraints,

$$d_t, m_t, b_t, a_t^i, x_{1t}, x_{2t} \geq 0 \tag{33}$$

The problem (28) subject to the constraints (29)-(33) states that a banking contract is chosen in equilibrium to maximize expected utility of the buyers subject to participation

[6] This extension is similar to Sanches and Williamson (2010) in which there are two types of meetings by the types of buyers in the *DM*. In a fraction ρ of *DM* meetings with type 1 buyers, fiat money are the only assets recognized by sellers. In $1 - \rho$ fraction of *DM* meetings with type 2 buyers, the sellers can receive the entire portfolio held by buyers.

[7] Note that this model can be reduced to the baseline model of Williamson (2012) if type information is perfect and $\theta = 1$.

constraint (29) in which banks receive a non-negative profit by providing the contract and cash constraint for type 1 buyers (30) and collateral constraint for type 2 buyers (31) and the incentive constraint for type 2 buyers (32) and non-negative constraints (33). I omitted the incentive constraint for type 1 buyers because it does not bind in equilibrium as shown in Lemma 1 and 2.[8] In (28)-(33),d_t denotes deposit of buyers, a_t denote demand for illiquid asset holdings of banks, ψ_t denote the prices of illiquid assets in the *CM*, m_t and b_t denote the quantities of money and government bonds in terms of the *CM* good in period t held by banks and x_{jt} denote the consumption of type j agents at time t *CM* for $j \in \{1, 2\}$. From now on I focus on stationary equilibrium where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all t, and μ is the gross inflation rate. Note that nominal interest rate of government bonds cannot be negative, i.e. $z_t \leq 1$, in equilibrium by its feasibility assumption. I assume that monetary authority sets the inflation rate target and implement OMOs to achieve its goal.

From the maximization problem the first-order conditions by $m_t, b_t, a_t, x_{1t}, x_{2t}$ can be described as follows.

$$\frac{\mu}{\beta} = 1 + \lambda_1 + \lambda_2 \tag{34}$$

$$z\frac{\mu}{\beta} = \frac{\psi}{\beta(\psi + y)} = 1 + \lambda_2 \tag{35}$$

$$u^{'}(x_1) - \frac{\lambda_3}{\rho} = 1 + \lambda_1 + \lambda_2 \tag{36}$$

$$\theta u^{'}(x_2) + \frac{\lambda_3}{1-\rho} = 1 + \lambda_2 \tag{37}$$

where $\lambda_1$to λ_3 denote each multiplier associated with the constraints (30)-(32). In equilibrium asset markets clear in the *CM* with

$$a^i = 1 \tag{38}$$

$$m = \phi M \tag{39}$$

$$b = \phi B \tag{40}$$

Since the supply of government assets are restricted by the consolidated government debt limit V we have

$$m + zb \leq V \tag{41}$$

Definition 3. *Given (ρ, V, y) and the inflation rate target μ, a stationary monetary equilibrium consists of quantities (x_1, x_2) and prices (z, ψ) and multipliers $(\lambda_1, \lambda_2, \lambda_3)$ which solve equations (30)-(32),(34)-(37),(41).*

Since quasi-linear utility is adopted the real return of assets such as fiat money, $\mu = \frac{\phi_{t-1}}{\phi_t}$, and government bonds, $\frac{1}{z\mu}$, illiquid Lucas tree, $\frac{\psi^i + y^i}{\psi^i}$, cannot exceed the rate of time preference, $\frac{1}{\beta}$. The rate of returns in government bonds and Lucas tree are same in equilibrium because there is no credit risk in Lucas tree. Nominal interest rate of government bonds

[8]Both individual incentive constraints do not change although type 2 buyers can trade since liquid and illiquid assets are used as same for collateral.

cannot be negative, i.e. $z \leq 1$, by its feasibility assumption. Then we have no arbitrage condition in equilibrium,

$$\frac{1}{\mu} \leq r \equiv \frac{1}{z\mu} = \frac{\psi^i + y^i}{\psi^i} \leq \frac{1}{\beta}. \tag{42}$$

Note that if the truth-telling constraint (32) binds then cash constraint (30) is relaxed because only one of them can restrict $x_1 \in [0, x^*)$ in equilibrium. Moreover, when the truth-telling constraint (32) binds collateral constraint (31) is required to bind, otherwise the truth-telling constraint (32) does not bind with $x_2 = x_2^*$. Hence when the truth-telling constraint (32) binds we have only one case of equilibrium with $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$. On the other hand, if the truth-telling constraint (32) does not bind then we have four cases of equilibrium with combination of λ_1 and λ_2. However, the case constraint (30) cannot be relaxed alone because the liquid assets can be also useful for credit arrangement. Thus we have totally four equilibrium cases.

(1) Friedman rule Equilibrium When all of the constraints (30)-(32) do not bind with $\lambda_1 = \lambda_2 = \lambda_3 = 0$, we have

$$\frac{\mu}{\beta} = z\frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \tag{43}$$

$$1 = u^{'}(x_1) = \theta u^{'}(x_2) \tag{44}$$

from the first-order conditions (34)-(37). Note that $x_1 = x_1^*$,$x_2 = x_2^*$ and $\psi^i = \psi^i_f$ where $u^{'}(x_1^*) = 1$, $\theta u^{'}(x_2^*) = 1$, $\psi^i_f = \frac{\beta y^i}{1-\beta}$ in equilibrium. Moreover, the rates of return in both liquid and illiquid assets are the same as the inverse of time preference, $\frac{1}{\mu} = r = \frac{1}{\beta}$ where $r := \frac{1}{z\mu} = \frac{\psi^i + y^i}{\psi^i}$ from (43)-(44) in equilibrium. Thus in this case the Friedman rule, $\mu = \beta$, is feasible so that the first-best allocation is achieved. The equilibrium is supported by a region with $V \geq \rho x_1^*$ and $V + \psi^i_f \geq \rho x_1^* + (1-\rho)x_2^*$ from the equations (30)-(31), (38), (41). Note that this case is the same as the case (i) in competitive equilibrium in the previous section.

(2) Currency-shortage Equilibrium Suppose that the Friedman rule equilibrium is infeasible with $V < \rho x_1^*$, while$\psi^i_f \geq (1-\rho)x_2^*$ is valid. With $\lambda_1 > 0$, $\lambda_2 = \lambda_3 = 0$ we have the first-order conditions,

$$\frac{\mu}{\beta} = u^{'}(x_1) \tag{45}$$

$$z\frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 = \theta u^{'}(x_2) \tag{46}$$

in which $x_2 = x_2^*$ and $\psi^i = \psi^i_f$. Binding cash constraint (30) is transformed into $\rho x_1 u^{'}(x_1) = V$. Thus $x_1 < x_1^*$ is fixed in equilibrium. Then in a region of $x_1 \in (0, x_1^*)$ we have $x_1 < x_2^*$and $\frac{1}{\mu} < r = \frac{1}{\beta}$ in equilibrium. Define this case as *Currency-shortage Equilibrium.* In this case real interest rate of illiquid asset is fixed and a liquidity premium arises in the price of money since only money is scarce. Open market operations are ineffective in real allocations since real interest rate of illiquid assets and x_1 are fixed in equilibrium. Note that this case is the same as the case (ii) in competitive equilibrium in the previous section.

(3) Asset-shortage Equilibrium Suppose that the Friedman rule equilibrium is infeasible with $V + \psi_f^i < \rho x_1^* + (1-\rho)x_2^*$, but $V \geq \rho x_1^*$ is still valid. Moreover, assume that $\lambda_3 = 0$. Then we have the first-order conditions,

$$\frac{\mu}{\beta} = u^{'}(x_1) \tag{47}$$

$$z\frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = \theta u^{'}(x_2) \tag{48}$$

from (34)-(37). Binding constraints (30) and (31) with asset market clearing condition (38) and government budget constraint (41) can be reduced into

$$\rho x_1 u^{'}(x_1) + (1-\rho)\theta x_2 u^{'}(x_2) = V + \frac{\beta y^i \theta u^{'}(x_2)}{1 - \beta\theta u^{'}(x_2)} \tag{49}$$

Given the price of government bonds z, x_1 and x_2 are positively related in (47) and (48) while x_1 and x_2 is negatively related in (49). Thus there exists a unique equilibrium allocation (x_1, x_2) as shown in Figure 6. Let me briefly describe threshold points as shown in Figure 6. There is a threshold point $\hat{x}_1$ at $x_2 = x_2^*$ in which the collateral constraint starts to bind. There is another point $\tilde{x}_1$ in which $x_1 = x_2$ holds with the nominal interest rate $z = \theta$. Finally, there is a point $\bar{x}_1$ where the equilibrium allocation is determined with the zero nominal interest rate, $z = 1$. Through open market operations, the monetary authority can inject money and absorb government bonds in the market. By conducting this procedure the currency trade x_1 increases whereas the credit arrangement x_2 decreases. Thus the monetary authority can choose an equilibrium allocation in $x_1 \in (\hat{x}_1, \bar{x}_1]$ by choosing the price of government bonds z in equilibrium. However, in this private information case the truth-telling constraint matters when x_1 becomes greater than x_2. Thus this asset-shortage equilibrium can exist only in $x_1 \in (\hat{x}_1, \tilde{x}_1]$ with $z \leq \theta$.

[Figure 6 here]

(4) Liquidity-trap Equilibrium Given the Friedman rule equilibrium is infeasible with $V + \psi_f^i < \rho x_1^* + (1-\rho)x_2^*$ and $V \geq \rho x_1^*$, suppose that truth-telling constraint (32) binds with $\lambda_3 > 0$. As discussed when (32) binds we have only one case of equilibrium with $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$. The first-order conditions are reduced into

$$\frac{\mu}{\beta} = z\frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = u^{'}(x_1) - \frac{\lambda_3}{\rho} = 1 + \lambda_2 \tag{50}$$

$$\theta u^{'}(x_2) + \frac{\lambda_3}{1-\rho} = 1 + \lambda_2 \tag{51}$$

from (34)-(37). Since the truth-telling constraint binds, we have $x \equiv x_1 = x_2$ in equilibrium. Thus (50) and (51) can be reduced into

$$u^{'}(x) - \frac{\lambda_3}{\rho} = \theta u^{'}(x) + \frac{\lambda_3}{1-\rho} \tag{52}$$

Then binding constraints (31)-(32) with asset market clearing condition (38) and government budget constraint (41) can be reduced into

$$\rho x u^{'}(x) + (1-\rho)\theta x u^{'}(x) = V + \frac{\beta y^i \{\rho u^{'}(x) + (1-\rho)\theta u^{'}(x)\}}{1 - \beta\{\rho u^{'}(x) + (1-\rho)\theta u^{'}(x)\}} \tag{53}$$

The allocation x is determined at $x = \breve{x}_1$ in which government budget constraint intersects with 45 degree line as shown in Figure 6. Note that $\breve{x}_1$ is fixed and $\frac{1}{\mu} = r < \frac{1}{\beta}$ holds with $z = 1$ in equilibrium. Thus in the *Liquidity-trap equilibrium* rates of return in money and illiquid assets are same and open market operations are no longer effective. Note that the feasibility condition (53) is similar to (49) with $x_1 = x_2$. However, the consumption level x in (53) is greater than the consumption level $x_1 = x_2$ in (49) with $z = \theta$ because liquid assets also have a liquidity premium in the liquidity trap equilibrium.

4.1 Liquidity Trap and Excess Reserves

In this subsection let me elaborate the implementation of monetary policy in the *Asset-shortage Equilibrium* and *Liquidity-trap equilibrium*. Given the scarcity of illiquid assets with $V + \psi_f^i < \rho x_1^* + (1-\rho)x_2^*$ and $V \geq \rho x_1^*$, there exist two different regions in the equilibrium under private information. In a region of $x_1 \in (\hat{x}_1, \tilde{x}_1]$ we have $x_1 < x_1^*$, $x_2 < x_2^*$and $\frac{1}{\mu} < r < \frac{1}{\beta}$ in the *Asset-shortage Equilibrium*. In equilibrium the monetary authority can choose the equilibrium allocation along with the feasibility condition (49) by exchanging outside currency and government bonds in the market. For example, injecting money and absorbing government bonds decreases the nominal interest rate, $\frac{1}{z} - 1$, and currency trade x_1 increases whereas credit arrangement $x_2$decreases. On the other hand, in a point of $x_1 = \breve{x}_1$ we have $x_1 < x_1^*$, $x_2 < x_2^*$and $\frac{1}{\mu} = r < \frac{1}{\beta}$ in the *Liquidity-trap Equilibrium* as described.

Thus if the type information is public, then the equilibrium allocation $x_1 = \bar{x}_1$ with $z = 1$ is feasible. Note that in this equilibrium we have $x_2 < x_1$ and $\frac{1}{\mu} = r < \frac{1}{\beta}$. Let's define this equilibrium case as *Zero-lower-bound(ZLB) Equilibrium*. In the *Zero-lower-bound equilibrium* rates of return in money and illiquid assets are also same as shown in *Liquidity-trap equilibrium*. Thus open market operations are also ineffective in real allocations. Injecting money just increases the amount of excess reserves and the nominal interest rate is zero in equilibrium. However, the equilibrium allocations are different between *Zero-lower-bound equilibrium* and *Liquidity-trap equilibrium*. Moreover, the reasons for the ineffective monetary policy are different. In the *Zero-lower-bound equilibrium*, monetary policy is ineffective because the rates of return are set as same for both liquid and illiquid assets. But in the *Liquidity-trap equilibrium* monetary policy is ineffective because the excess reserves are required to separate the types under private information. Thus the same rates of return on liquid and illiquid assets with zero nominal interest rate is a consequence of equilibrium allocation instead of choice of monetary authority.

Proposition 2. *Given $\theta < 1$, there exists a unique Liquidity-trap equilibrium away from the Zero-lower-bound equilibrium under private information.*

Proof. *In Liquidity-trap equilibrium the allocation $x = x_1 = x_2$ which satisfies (53). It is unique since the left side of (53) is strictly increasing in x while the right side of (5) is strictly decreasing in x. The allocation $x = x_1 = x_2$ which satisfies (53) is different from the allocation in the Zero-lower-bound equilibrium because $x_1 > x_2$ holds in the Zero-lower-bound equilibrium. QED.*

Note that if $\theta = 1$ is assumed, *Liquidity-trap equilibrium* overlaps with *Zero-lower-bound equilibrium* because the truth-telling constraint does not bind even at the zero nominal interest rate. It is hard to differentiate the existence of *Liquidity-trap equilibrium* from *Zero-lower-bound equilibrium*in reality. However, if there exists a cost of operating credit arrangement or inefficiency in credit arrangement such as haircut then there could exist a jump from $z = \theta$ to $z = 1$ in equilibrium.[9] Figure 7, which is replicated from Orphanides (2004), describes a movement of nominal interest rates along with excess reserves in the period of Great Depression. It is shown that there exists a volatile movement in nominal interest rates with excess reserves in a period of Great Depression. This implies at least that the monetary authority could lose its control on nominal interest rates in a neighborhood of zero lower bound.

[Figure 6 here]

5 Conclusion

In the paper I construct a banking model to study how private information confines liquidity insurance and the implementation of monetary policy. Given idiosyncratic liquidity shocks lack of record-keeping assumption generates private information on types. A truth-telling banking contract is offered to provide liquidity efficiently under private information. When the supply of total assets are not enough to support liquidity distribution, the truth-telling incentive constraint binds and a liquidity premium arises in the price of illiquid assets to reveal private information. In the extended model with monetary policy when the truth-telling constraint binds there exists a liquidity trap in which OMOs is ineffective in real allocations. This *Liquidity-trap equilibrium* is different with the previous ones with currency-shortage or zero-lower-bound because it is generated by the incentive of banks to hold illiquid and even liquid assets for efficient liquidity provision.

This paper takes a step forward to understand liquidity trap. It provides a model in which liquidity trap can exist when banks have an incentive to hold liquid assets in their balance sheets so that it opens a possibility to study further on liquidity trap. However, it also leaves further questions unanswered. For example, liquid assets in bank's balance sheet can play a role to prevent bank runs. In this respect we can ask how fragility of banks are associated with the effectiveness of monetary policy. This requires a deeper consideration and explicit modeling on bank runs to study further.

[9] However, if a floor system which directly sets interest rates for reserves is available, then nominal interest rates are achieved exactly by setting the interest on reserves as same as the nominal interest rate target. Thus there would exist no jump in nominal interest rates.

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Figure 1. Federal Fund Rates and Excess Reserves in 2008

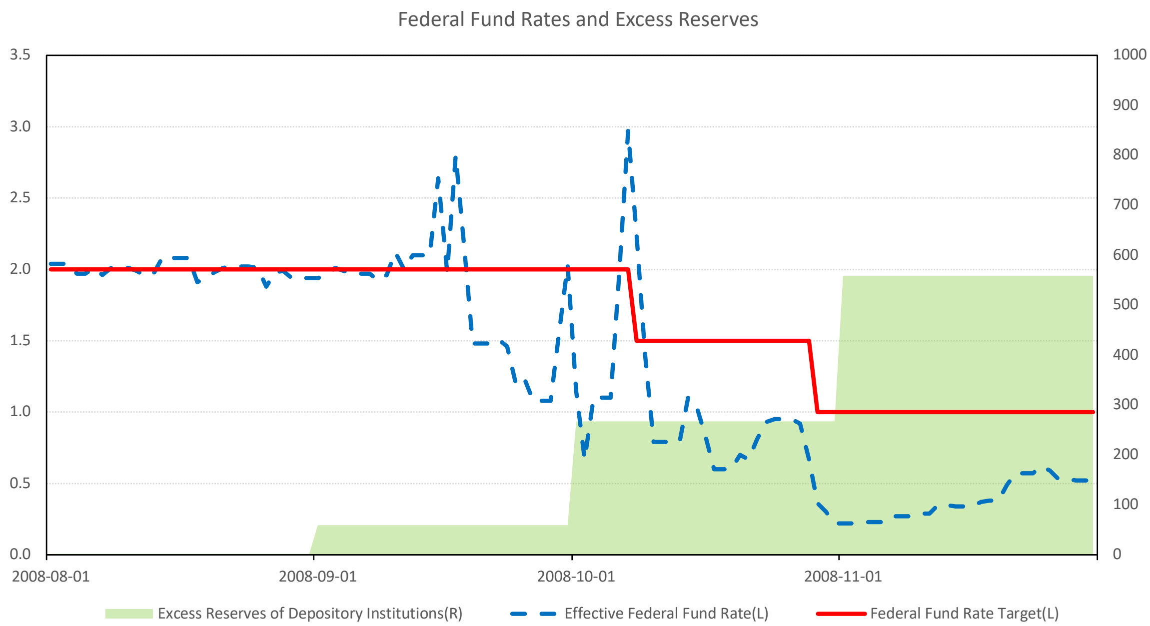


Figure 2. Timing

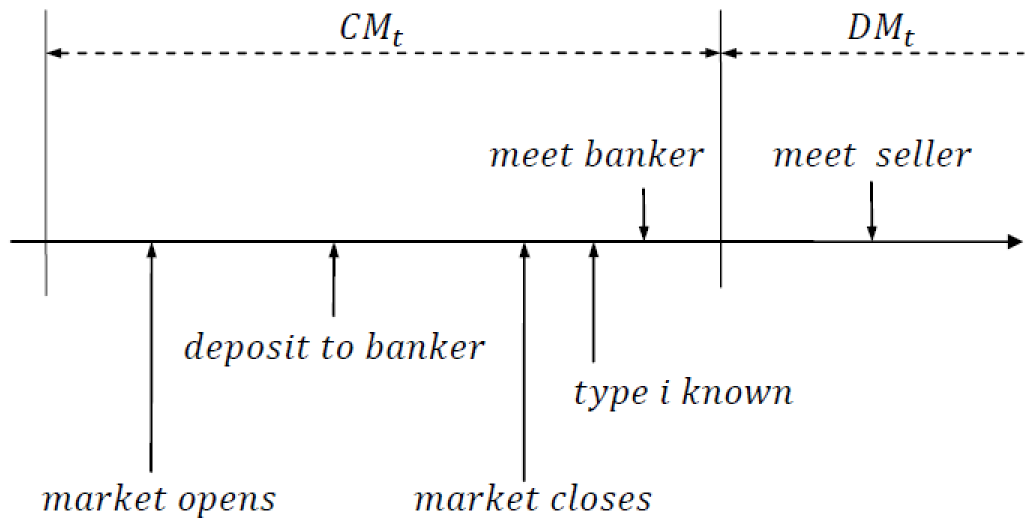


Figure 3. Perfect Information

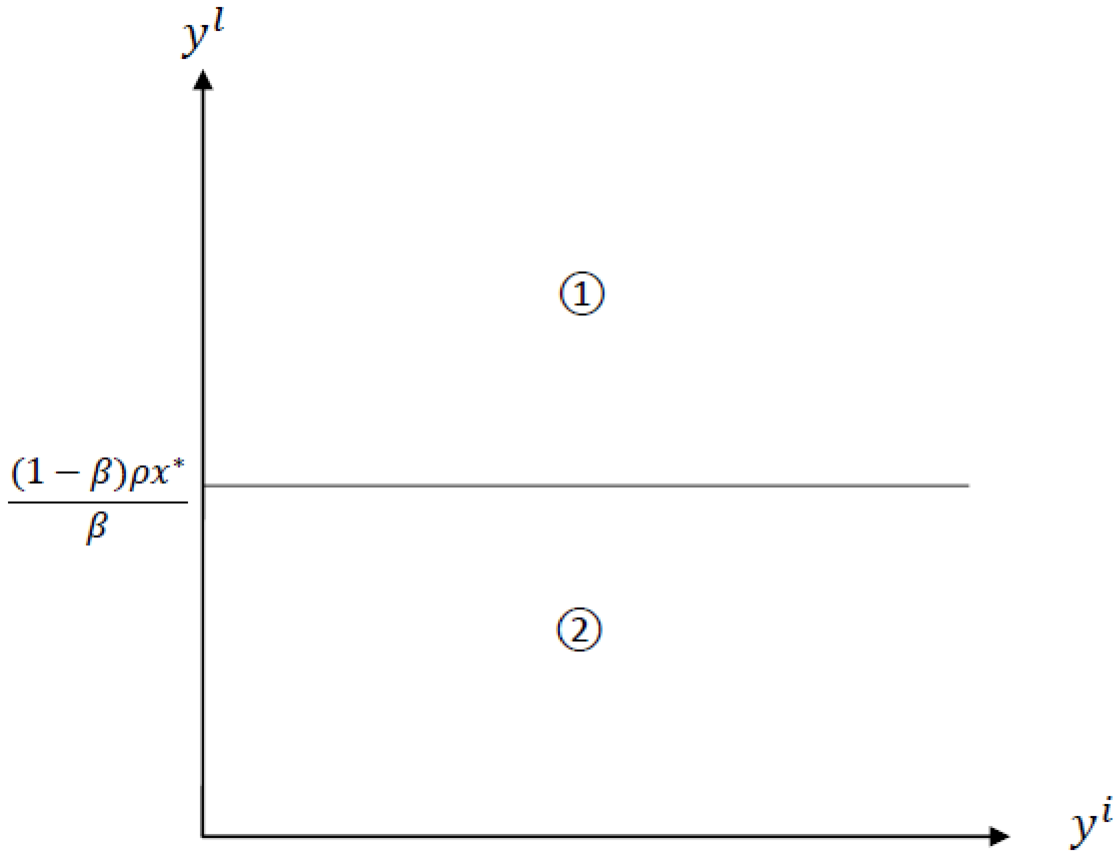


Figure 4. Single Crossing Property

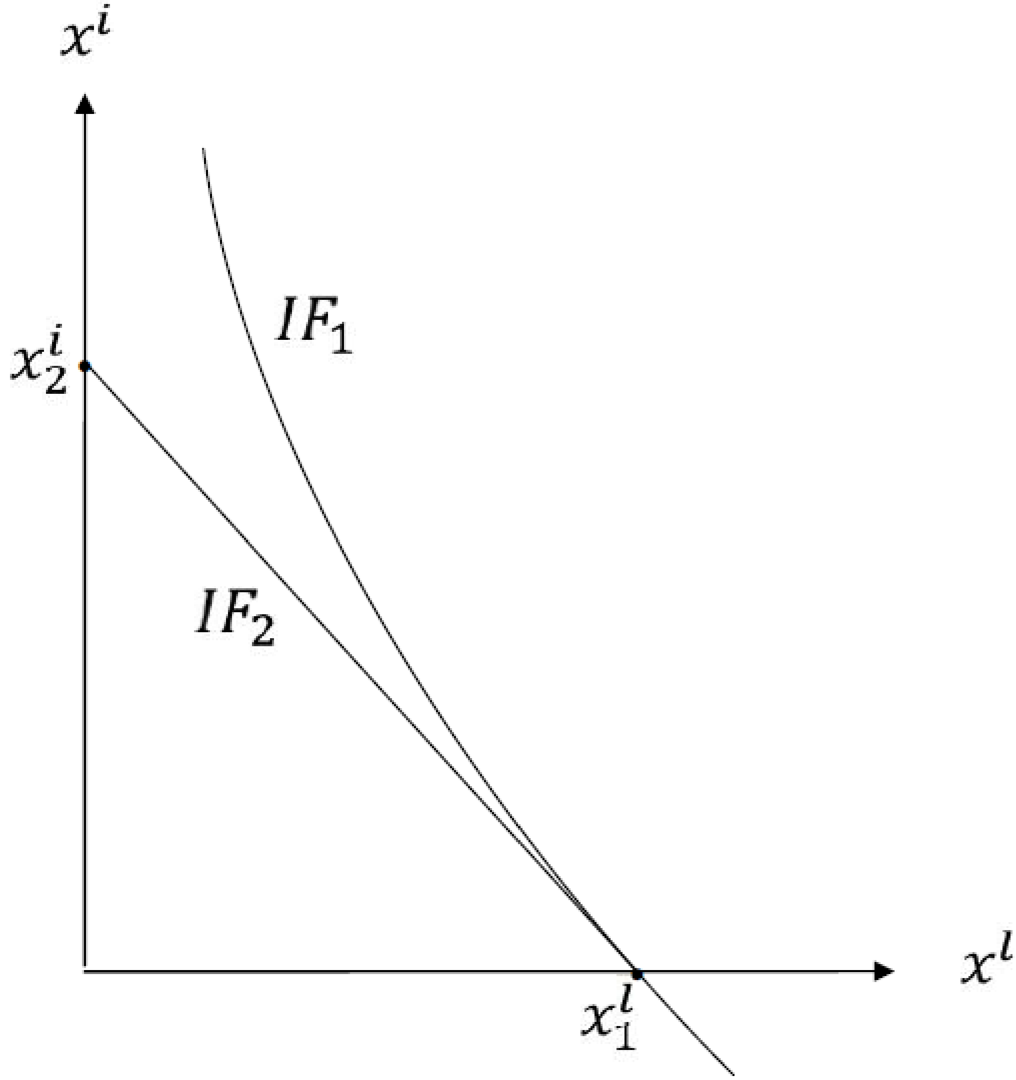


Figure 5. Private Information

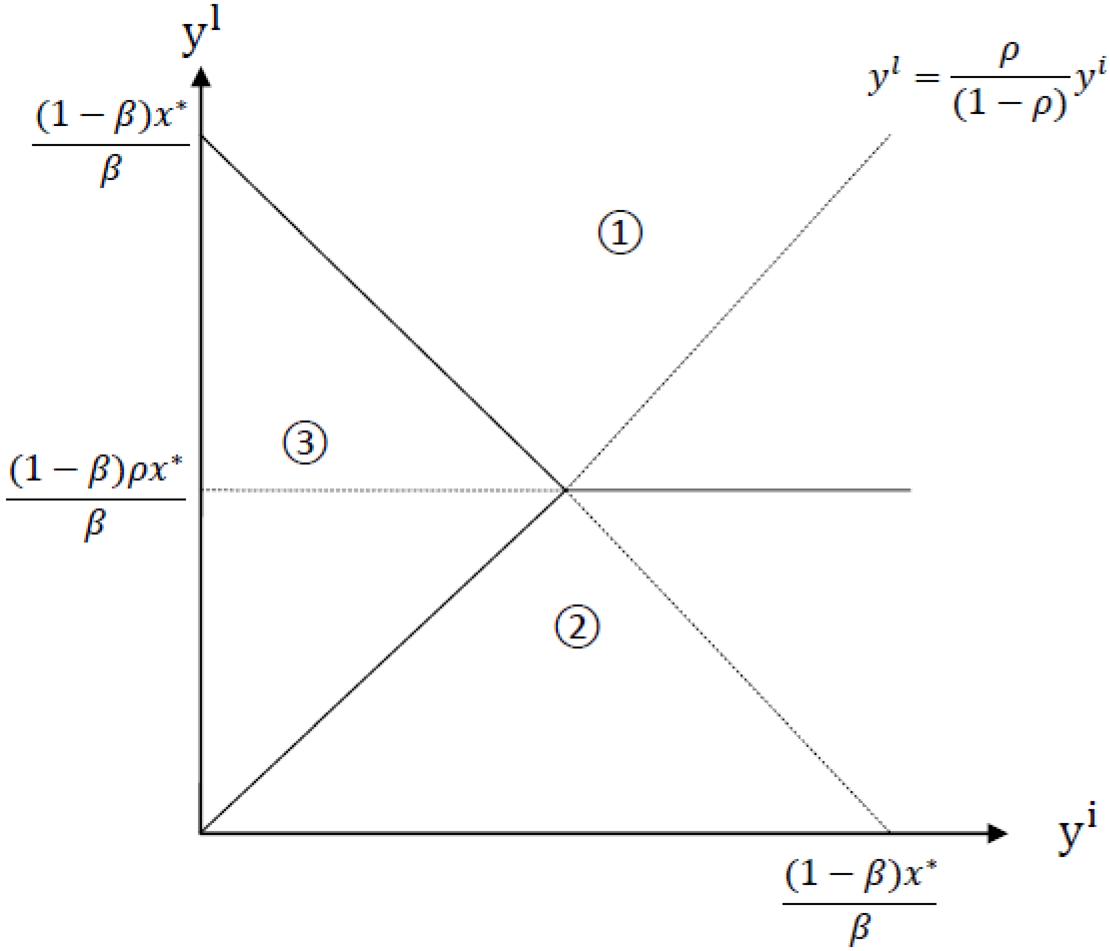


Figure 6. Liquidity Trap Equilibrium

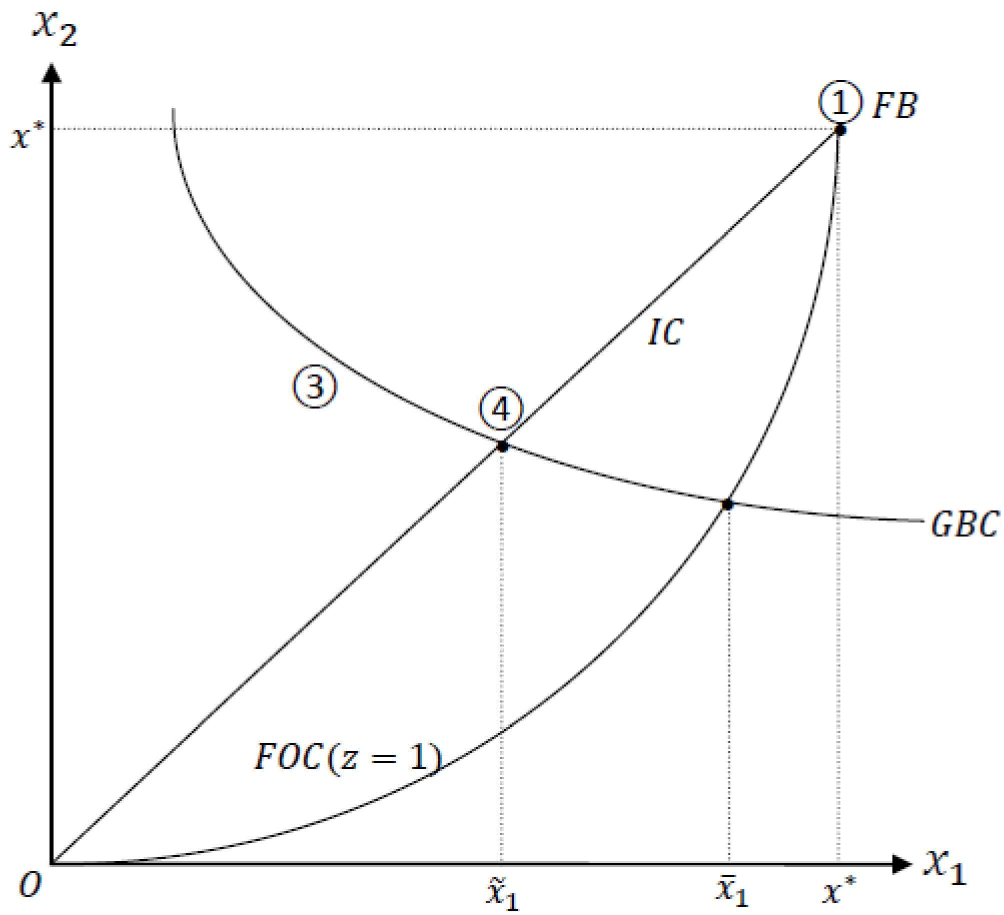
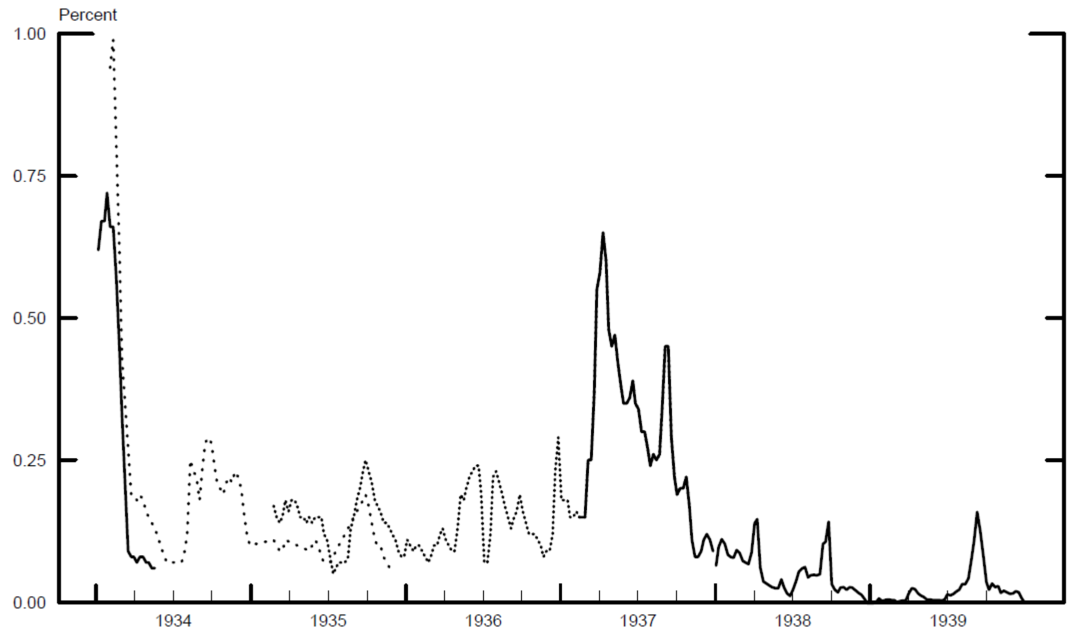


Figure 7. Treasury Bill Rates and Excess Reserves in 1930s

Treasury Bill Rates



Excess Reserves